An Upper Limit on the Neutrino Rest Mass*

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In order that the effect of graviation of the thermal background neutrinos on the expansion of the universe not be too severe, their mass should be less than 8 eV/c^2.

Recently there has been a resurgence of interest in the possibility that neutrinos may have a finite rest mass. These discussions have been in the context of weak-interaction theories,[1] possible decay of solar neutrinos,[2] and enumerating the possible decay modes of the $K_L{}^0$ meson.[3] Elsewhere, we have pointed out that the gravitational interactions of neutrinos of finite rest mass may become very important in the discussion of the dynamics of clusters of galaxies and of the universe.[4] Considerations involving massive neutrinos are not new[5,6]; an excellent review of the early developments in the field is given by Kuchowicz.[7] Here we wish to point out that the recent measurement[8] of the deceleration parameter, q_0, implies an upper limit of a few tens of electron volts on the sum of the masses of all the possible light, stable particles that interact only weakly.

In discussing this problem we take the customary point of view that the universe is expanding from an initially hot and condensed state as envisaged in the "big-bang" theories.[9] In the early phase of such a universe, when the temperature was greater than ~1 MeV, processes of neutrino production, which have also been considered in the context of high-temperature stellar cores,[10] would lead to the generation of the various kinds of neutrinos. In fact, similar processes would generate populations of other fermions and bosons as well, and conditions of thermal equilibrium allow us to estimate their number density[11]:

$$n_{Fi} = \frac{2s_i+1}{2\pi^2\hbar^3}\int_0^\infty \frac{p^2dp}{\exp[E/kT(z_{eq})]+1}, \tag{1a}$$

and

$$n_{Bi} = \frac{2s_i+1}{2\pi^2\hbar^3}\int_0^\infty \frac{p^2dp}{\exp[E/kT(z_{eq})]-1}. \tag{1b}$$

Here n_{Fi} is the number density of fermions of the ith kind, n_{Bi} is the number density of bosons of the ith kind, s_i is the spin of the particle (notice that in writing the multiplicity of states of the particles we have not discriminated against the neutrinos; since we are discussing neutrinos of nonzero rest mass, we have assumed that both the helicity states are allowed), $E = c(p^2+m^2c^2)^{1/2}$, k is Boltzmann's constant, and $T(z_{eq}) = T_r(z_{eq}) = T_F(z_{eq}) = T_B(z_{eq}) = T_m(z_{eq}) = \cdots$ is the common temperature of radiation, fermions, bosons, matter, etc. at the latest epoch, characterized by the red shift z_{eq}, when they may be considered to be in thermal equilibrium; $kT(z_{eq}) \approx 1$ MeV.

Since our discussion pertains to neutrinos and any hypothetical stable weak bosons,[2] we may assume that $kT(z_{eq}) \approx 1\text{ MeV} \gg mc^2$. In this limit Eqs. (1a) and (1b) reduce to

$$n_F(z_{eq}) \approx 0.0913(2s_i+1)[T(z_{eq})/\hbar c]^3, \tag{2a}$$

$$n_B(z_{eq}) \approx 0.122(2s_i+1)[T(z_{eq})/\hbar c]^3. \tag{2b}$$

As the universe expands and cools down, the neutrinos and such other weakly interacting particles survive without annihilation because of the extremely low cross sections[12] for these processes. Consequently, the number density decreases simply as $\sim V(z_{eq})/V(z) = (1+z)^3/(1+z_{eq})^3$. Noticing that $1+z = T_r(z)/T_r(0)$, the number densities of the various particles expected at the present epoch ($z=0$) are given by

$$n_{Fi}(0) = n_{Fi}(z_{eq})\left[\frac{1}{1+z_{eq}}\right]^3 \approx 0.0913(2s_i+1)\left[\frac{T_r(0)}{\hbar c}\right]^3 \tag{3a}$$

and

$$n_{Bi}(0) \approx 0.122(2s_i+1)\left[\frac{T_r(0)}{\hbar c}\right]^3. \tag{3b}$$

Taking $T_r(0) \approx 2.7°$K, we have

$$n_{Fi}(0) \approx 150(2s_i+1)\text{ cm}^{-3}, \tag{4a}$$

and

$$n_{Bi}(0) \approx 200(2s_i+1)\ \text{cm}^{-3}. \tag{4b}$$

These numbers are huge in comparison with the mean number density of hydrogen atoms in the universe; all the visible matter in the universe adds up to an average density of hydrogen atoms[9] of only $\sim 2\times 10^{-8}$ cm^{-3}. Notice that the expected density of the neutrinos and other weakly interacting particles is essentially independent of the temperature $T(z_{eq})$, of decoupling, and such other details; the measured temperature of the universal blackbody photons fixes the density of weak particles quite well.

Now, consider Sandage's[8] measurement of the Hubble constant H_0 and the deceleration parameter q_0 which together place a limit on ρ_{tot}, the density of all possible sources of gravitational potential in the universe.[9] His results, $H_0 = 50$ km sec^{-1} Mpc^{-1} $= 1.7\times 10^{-18}$ sec^{-1} and $q_0 = +0.94 \pm 0.4$, imply

$$\rho_{\text{tot}} = 3H_0^2 q_0/4\pi G = (10\pm 4)\times 10^{-30}\ \text{g cm}^{-3} \approx (6\pm 2)\times 10^3\ (\text{eV}/c^2)\ \text{cm}^{-3} < 10^4\ (\text{eV}/c^2)\ \text{cm}^{-3}. \tag{5}$$

Here $G = 6.68\times 10^{-8}$ dyn cm^2 g^{-2} is the gravitational constant. If m_i were to represent the mass spectrum of the various neutrinos and other stable weakly interacting particles, we can combine Eqs. (4a), (4b), and (5) to obtain the limit

$$\rho_{\text{weak}} \approx \sum n_{Bi} m_i + n_{Fj} m_j \gtrsim 150(2s_i+1)m_i < \rho_{\text{tot}}$$

or (6)

$$\sum (2s_i+1)m_i \lesssim 66\ \text{eV}/c^2.$$

Here the summation is to be carried out over all the particle and antiparticle states of both fermions and bosons. Considering only the neutrinos and antineutrinos of the muon and electron kind each having a mass of m_ν, Eq. (6) leads to the result $m_\nu < 8$ eV/c^2.

This limit is obtained assuming big-bang cosmology to be correct; however, it depends only very weakly on the value of the deceleration parameter and other details of the cosmology. Thus, even when one allows for a large uncertainty in the cosmological parameters, the limits on the masses of neutrinos and other stable weakly interacting particles derived in this paper are still much lower than the direct experimental limits[13,14] of $m_{\nu\mu} < 1.5$ MeV/c^2 and $m_{\nu e} < 60$ eV/c^2.

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[1]K. Tennakone and S. Pakvasa, Phys. Rev. Lett. 27, 757 (1971), and 28, 1415 (1972).

[2]J. N. Bahcall, N. Cabibbo, and A. Yahil, Phys. Rev. Lett. 28, 316 (1972).

[3]S. Barshay, Phys. Rev. Lett. 28, 1008 (1972).

[4]R. Cowsik and J. McClelland, "Gravity of Neutrinos of Non-Zero Mass in Astrophysics" (to be published).

[5]M. A. Markov, *The Neutrino* (Nauka, Moscow, U. S. S. R., 1964).

[6]J. Bahcall and R. B. Curtis, Nuovo Cimento 21, 422 (1961).

[7]B. Kuchowics, *The Bibliography of the Neutrino* (Gordon and Breach, New York, 1967), and Fortschr. Phys. 17, 517 (1969).

[8]A. Sandage, Astrophys. J. 173, 485 (1972), and to be published.

[9]P. J. E. Peebles, *Physical Cosmology* (Princeton Univ. Press, Princeton, N. J., 1971).

[10]M. A. Ruderman, in *Topical Conference on Weak Interactions, CERN, Geneva, Switzerland, 1969* (CERN Scientific Information Service, Geneva, Switzerland, 1969), p. 111.

[11]L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Addison-Wesley, Reading, Mass., 1969), 2nd ed., p. 324.

[12]T. de Graff and H. A. Tolhoek, Nucl. Phys. 81, 596 (1966).

[13]K. Bergkvist, Nucl. Phys. B39, 317 (1972).

[14]E. V. Shrum and K. O. H. Ziock, Phys. Lett. 37B, 115 (1971).