LIMITS ON MASSES AND NUMBER OF NEUTRAL WEAKLY INTERACTING PARTICLES

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Limits on the masses and number of neutral weakly interacting particles are derived using cosmological arguments. No such particles with a mass between 120 eV and 3 GeV can exist within the usual big band model. Similar, but much more severe, restrictions follow for particles that interact only gravitationally. This seems of importance with respect to supersymmetric theories.

Following an idea, put forward by Shvartsman [1], Steigman et al. [2] presented arguments leading to an upper limit to the number of different types of massless neutrinos, which may be summarized as follows.

According to the hot big bang model all forms of matter in the universe, even neutrinos, are initially in thermal equilibrium. The total energy density of relativistic particles is then given at a temperature T by

$$\rho = \kappa a T^4 . \tag{1}$$

a is the radiation density constant, appearing in the black-body radiation law, and κ is given by

$$\kappa = \tfrac{1}{2}(n_b + \tfrac{7}{8}\, n_f). \tag{2}$$

The quantities n_b and n_f are the total number of internal degrees of freedom of the different types of bosons and fermions respectively. For a photon gas $\kappa = 1$, while for a mixture of photons, electrons, electron and muon neutrinos, together with their antiparticles, $\kappa = 9/2$.

A second expression for the total energy density ρ is given as a function of the expansion time t by solving the Einstein equations in a radiation dominated homogeneous and isotropic universe,

$$\rho = 3/32\, \pi G t^2 , \tag{3}$$

where G is the gravitational coupling constant, $G = 6.7 \times 10^{-45}\ \mathrm{MeV}^{-2}$‡. Combining (1) and (3) we get

$$T = (3/32\, \pi G a)^{1/4}\, \kappa^{-1/4}\, t^{-1/2}. \tag{4}$$

‡ We use units such that $\hbar = c = k = 1$, and the temperature is expressed in MeV.

Adding more types of neutrinos relative to the standard big bang model increases the value of κ. This would have the following observable effect.

The neutron/proton ratio is given by the equilibrium value $n/p = \exp\{-(m_n - m_p)/T\}$ as long as the rate of weak interactions, like e.g. $n + e^+ \leftrightarrows p + \bar{\nu}_e$, is high enough. But this ratio freezes in soon after the time between successive collisions grows bigger than, say, the expansion time. The mean free time is $\tau = (\sigma N)^{-1}$ as long as the electrons are relativistic. The cross section $\sigma \sim T^2$ and the number density of protons and neutrons $N \sim R^{-3}$, where R is the scale factor of the expanding universe. At these early times the number of nucleons is far smaller than the number of photons, electrons, positrons and neutrinos, so the cooling proceeds adiabatically like $T \sim R^{-1}$. Therefore $N \sim T^3$ and thus

$$\tau = \text{const.} \times T^{-5}. \tag{5}$$

Putting $t = \tau$ in (4), from (5) we get an effective temperature T_f at which the neutron/proton ratio freezes in, given by

$$T_f = \text{const.} \times \kappa^{1/6}. \tag{6}$$

When the temperature falls off further nearly all neutrons are captured to form deuterium and subsequently helium. In the standard model $T_f \approx 1\ \text{MeV} \approx 10^{10}$ K and the abundance by weight of helium produced in this way is $Y \approx 0.23$ to 0.27, depending on the present density of nucleons in the universe. An observational upper limit [4] $Y \lesssim 0.29$ agrees well with the standard model.

Increasing now the number of neutrino types would increase κ and via (6), T_f. Therefore the neutron/proton ratio would freeze in at a higher value, and more helium would be formed. Since there is not much room left between the observational upper limit to Y and the predictions of the standard model, rather strong restrictions can be posed on κ. Steigman et al. [2], using numerical calculations by Wagoner [3], find $\kappa \lesssim 9$, twice the standard value $\kappa = 9/2$ at temperatures around $T_f \approx 1$ MeV ($m_\mu > T_f > m_e$). Assuming a doublet structure for heavy leptons they concluded from (2) that the total number of heavy leptons should be less than or equal to 5.

This argument can be extended and combined with arguments concerning the present matter density of the universe to apply to massive neutral weakly interacting particles in general, to be called massive neutrinos for short. No detailed assumption about the multiplet structure of which such neutrinos would be part is needed in this context. Also their spin is not essential, only the number of degrees of freedom is relevant.

Suppose now the existence of new neutrinos with a mass less than 1 MeV. Then these particles would be copiously present till the temperature dropped to $T \approx 1$ MeV. This is also the temperature at which the neutron/proton ratio freezes in, more specifically the temperature at which the universe becomes transparent for weak processes. Therefore neutrinos having a mass less than 1 MeV are as effective in the previous argument concerning the helium abundance as are massless neutrinos.

Neutrinos more massive than 1 MeV disappear by annihilation before the time t_f when the neutron/proton ratio freezes in. Therefore no observational consequences arise from such neutrinos.

The argument concerning the observed helium abundance applies also to unstable neutrinos provided their lifetime exceeds 1 second, which is roughly the mean free time at $T \approx 1$ MeV. Otherwise they do not play a significant role in the equilibrium situation.

Further restrictions on the number of stable neutrino types are imposed by the present lifetime of the universe, being 10^{10} years or more. Together with the observed expansion rate of the universe this puts an upper limit on the mean mass density of about 10^{-29} g/cm^3. Since for highly relativistic particles the energy density varies like $\rho \sim T^4$, as opposed to nonrelativistic particles for which $\rho \sim T^3$, the universe is initially dominated by radiation until the temperature drops to $T \approx 10^{-7} - 10^{-5}$ MeV (depending on the present mass density). Hereafter the energy density of the different forms of radiation left drops below the density of massive particles. If there would exist a stable kind of neutrino having a mass in the keV region, these particles would be present copiously at the time when the universe became transparent for weak interactions. Now they cool by expansion and they would soon become nonrelativistic. Therefore at present they would have a mass density far greater than the upper limit posed by the lifetime of the universe. Several authors have used this argument to find an upper limit to the rest mass of electron and muon neutrinos [5].

For light stable neutrinos an upper limit to the mass is computed in this way by Szalay and Marx [5]. The result is 30 eV in the case of electron or muon neutrinos, assuming two degrees of freedom per particle. Allowing a particle with only one degree of freedom which is its own antiparticle leads thus to an upper limit of 120 eV. At first sight this seems to imply for a particle with four degrees of freedom, accompanied by an antiparticle, a mass limit of 15 eV, but actually this value is about 1.5 eV. The reason is that the observed helium abundance is only compatible with eight extra degrees of freedom if the universe contains a mass density of at most 1/10 of the critical mass density [2]. These results are drawn in fig. 1 (full line).

For heavy stable neutrinos a lower limit to the mass m can be obtained by the following argument. When the temperature drops to $T \approx m$, most of the heavier particles have vanished already. Therefore the neutrinos can disappear only by annihilation with their antiparticles, and we have to estimate the temperature $T_f(m)$ at which annihilation effectively stops.

Suppose for simplicity that the neutrinos are in thermal equilibrium till the temperature drops to $T_f(m)$ and that they are completely free afterwards. This temperature can again be estimated by equating the mean free time τ between reactions involving only these heavy neutrinos and their antiparticles with the expansion time of the universe. For massless neutrinos it follows from (4) and (5) that

$$t/\tau = [T/1 \text{ MeV}]^3, \qquad (7)$$

where the constant of proportionality is determined by the temperature $T \approx 1$ MeV when the universe becomes transparent for weak interactions. For heavy

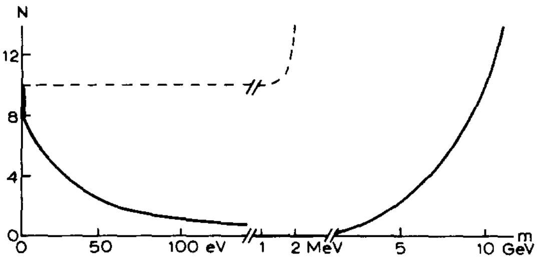

Fig. 1. Upper limits to the sum of the number of the internal degrees of freedom of the different types of neutral weakly interacting particles, apart from the electron and muon neutrinos The full line applies to stable particles; the dashed line to unstable particles if their halflife is more than 1 second. m denotes the mass of the particles, while N gives the number of degrees of freedom for fermions. For bosons, multiply N by a factor 7/8.

neutrinos, however, the energy density $\rho_L(m)$ compared to that of electron neutrinos at $T = T_f$ is approximately

$$\rho_L(m)/\rho_\nu = \tfrac{1}{2} \exp(-m/T_f(m)). \tag{8}$$

We assume here that the heavy neutrinos have only one degree of freedom and are their own antiparticle. Therefore the reaction rate τ^{-1} gets this extra dilution factor, and (7) goes over into

$$t/\tau = \tfrac{1}{2}[T_f(m)/1\ \mathrm{MeV}]^3 \exp(-m/T_f(m)) \equiv 1, \tag{9}$$

or

$$m = 3\,T_f(m)\,\{\ln T_f(m) - 0.23\}\,. \tag{10}$$

Now we must estimate the influence of the left-over neutrinos on the present mass density, which must obey

$$\rho_{L0} < 10^{-29}\,\mathrm{g/cm^3} \approx 5 \times 10^{-3}\ \mathrm{MeV/cm^3}. \tag{11}$$

According to the standard model the present temperature $T_{\nu 0}$ of electron neutrinos is related to that of the photons by $T_{\nu 0} = (4/11)^{1/3}\, T_{\gamma 0} = 1.7 \times 10^{-10}$ MeV, where the observed background radiation temperature $T_{\gamma 0} = 2.7\ \mathrm{K} = 2.3 \times 10^{-10}$ MeV. The present energy density of electron neutrinos is thus given by

$$\rho_{\nu 0} = 1.1 \times 10^{-34}\ \mathrm{g/cm^3} = 6.2 \times 10^{-8}\ \mathrm{MeV/cm^3}, \tag{12}$$

resulting in the inequality

$$\rho_{L0}/\rho_{\nu 0} < 10^5. \tag{13}$$

The energy density of the heavy neutrinos falls off like $\rho_L \sim T^3$, since they are already nonrelativistic at $T = T_f$, while $\rho_\nu \sim T^4$. From (13) it follows that at $T = T_f$

$$\frac{\rho_L(m)}{\rho_\nu} = \frac{T_{\nu 0}}{T_f(m)} \cdot \frac{\rho_{L0}}{\rho_{\nu 0}} < 2[T_f(m)/10^{-5}\ \mathrm{MeV}]^{-1}. \tag{14}$$

Comparing this with (8) and (10) gives

$$m > 3000\ \mathrm{MeV} \tag{15}$$

for the mass of a heavy neutrino with one degree of freedom. Similar calculations for more degrees of freedom are easily done and result in the full line in fig. 1.

In summary, stable massive neutral leptons cannot exist in the mass range 120 eV – 3000 MeV. Below 60 eV at most 10 degrees of freedom are available for fermions. For instance, only two types of neutrinos are allowed if they have two spin states per particle and antiparticle. Bosons have a weight factor of 8/7 with respect to fermions. Above 3000 MeV the allowed number rises quickly (see fig. 1).

Less stringent upper limits can be put on the number of unstable massive neutrino types. If their lifetime is more than 1 second the dashed line in fig. 1 applies, but for much shorter lifetimes no interesting limits can be obtained.

Finally the same arguments can be applied to particles which interact only gravitationally. For them the universe becomes transparent already at a much earlier time, estimated by Matzner [6] to be 10^{-16} sec. at a temperature of 10^{20} K $\approx 10^{10}$ MeV. These particles thus have a forbidden mass range of 60 eV – 10^{19} MeV. The upper limit of this range is still well below the Planck mass, being about 10^{22} MeV. Of course this limit is less reliable than the upper limit found for weakly interacting particles since not much is known for such early times after the big bang. But at least it gives a strong indication that the upper limit is very much more than, say, 1 TeV. Assuming the standard big bang model to be of relevance in the context of supergravity theories, one can make the following remark. If there exist light massive spin 3/2 particles interacting only gravitationally, having four spin degrees of freedom, their mass must be less than 15 eV if they are their own antiparticles, otherwise their mass is less than 1.5 eV. Also they may exist with masses very much larger than 1 TeV.

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