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Cosmological Lower Bound on Heavy Neutrino Masses

BENJAMIN W. LEE *
Fermi National Accelerator Laboratory, Batavia, Illinois 60510

AND

STEVEN WEINBERG **
Stanford University, Physics Department, Stanford, California 94305

ABSTRACT

The present cosmic mass density of possible stable neutral heavy leptons is calculated in a standard cosmological model. In order for this density not to exceed the upper limit of $2 \times 10^{-29} \text{ g/cm}^3$, the lepton mass would have to be greater than a lower bound of the order of 2 GeV.

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There is a well-known cosmological argument¹ against the existence of neutrino masses greater than about 40 eV. In the "standard" big-bang cosmology,² the present number density of each kind of neutrino is expected³ to be 6/11 the number density of photons in the 3°K black-body background radiation, or about 300 cm⁻³; hence if the neutrino mass were above 40 eV, their mass density would be greater than 2×10⁻²⁹ gm/cm³, which is roughly the upper limit allowed by present estimates⁴ of the Hubble constant and the deceleration parameter.

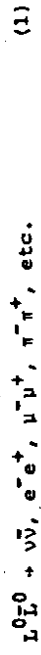
However, this argument would not apply if the neutrino mass were much larger than 1 MeV. Neutrinos are generally expected² to go out of thermal equilibrium when the temperature drops to about 10¹⁰°K, the temperature at which neutrino collision rates become comparable to the expansion rate of the universe. If neutrinos were much heavier than 1 MeV, then they would already be much rarer than photons at the time when they go out of thermal equilibrium, and hence their number density would now be much less than 300 cm⁻³.

Of course, the familiar electronic and muonic neutrinos are known to be lighter than 1 MeV. However, heavier stable neutral leptons could easily have escaped detection, and are even required in some gauge models.⁵ In this article, we suppose that there exists a neutral lepton L⁰ (the "heavy neutrino") with mass well above 1 MeV, and we assume that L⁰ carries some additive or multiplicative quantum number which keeps it absolutely stable. We will present arguments based on the standard big-bang cosmology to show that the mass of

such a particle must be above a lower bound of order 2 GeV.

At first glance, it might be thought that the present number density of heavy neutrinos would simply be less than the above estimate of 300 cm⁻³ by the value exp(-m_L/1 MeV) of the Boltzmann factor at the time the heavy neutrinos go out of thermal equilibrium. If this were the case, then an upper limit of 2×10⁻²⁹ g/cm⁻³ on the present cosmic mass density would require that m_Lexp(-m_L/1 MeV) should be less than 40 eV, and hence that m_L should either be less than 40 eV or greater than 13 MeV.

However, the true lower bound on the heavy neutrino mass is considerably more stringent. The heavy neutrinos are assumed to carry a conserved quantum number, so their number density can relax to its equilibrium value n₀ only by annihilation of a heavy neutrino with a heavy antineutrino, in processes such as



But although the energy distribution of the heavy neutrinos is kept thermalized by collisions with ν, e, etc. down to a temperature of 10¹⁰°K, at that temperature they are so rare that their annihilation rate is already much less than the cosmic expansion rate. Thus the heavy neutrinos go out of chemical equilibrium, (in the sense that their number density begins to exceed its equilibrium value) at a "freezing" temperature T_f which is much higher than 1 MeV. The condition on m_L is then that m_Lexp(-m_L/kT_f) should be less than 40 eV, and the resulting lower bound on m_L must therefore be greater than 13 MeV.

To make this quantitative, we use the rate equation

$$\frac{dn}{dt} = -\frac{3R}{R} n - \langle \sigma v \rangle n^2 + \langle \sigma v \rangle n_0^2 \quad (2)$$

Here n is the actual number density of heavy neutrinos at time t ; R is the cosmic scale factor; $\langle \sigma v \rangle$ is the average value of the $L\bar{L}^0$ annihilation cross-section times the relative velocity; and n_0 is the number density of heavy neutrinos in thermal (and chemical) equilibrium⁶.

$$n_0(T) = \frac{2}{(2\pi)^3} \int_0^\infty 4\pi p^2 dp \left[\exp\left(\frac{(m_L^2 + p^2)^{1/2}}{kT}\right) + 1 \right]^{-1} \quad (3)$$

(We use units with $\hbar=c=1$ throughout.)

At the temperatures we are considering here, the energy density and the entropy are dominated by highly relativistic particles, including γ, ν_e, ν_μ , and e . It follows that R, T and t are related by

$$\dot{R}/R = -\dot{T}/T = (8\pi\rho G/3)^{1/4} \quad (4)$$

where ρ is the energy density

$$\rho = N_F a T^4 = N_F \pi^2 (kT)^4 / 15 \quad (5)$$

with N_F an effective number of degrees of freedom, counting k and $7/16$ respectively for each boson or fermion species and spin state. For temperatures in the range of 10-100 MeV (which most concern us here) we must include just $\gamma, \nu_e, \nu_\mu, \bar{\nu}_e, \bar{\nu}_\mu, e^-,$ and e^+ , so $N_F = 4.5$, a value we will adopt for most purposes. However, if current ideas about the strong interactions are correct, then N_F rises steeply at a temperature of order 500 MeV to a value⁷ $N_F \approx 30$.

To estimate $\langle \sigma v \rangle$, we note that the heavy neutrinos must be quite non-relativistic at the temperature T_f where they freeze

out of chemical equilibrium, because as we have seen, the upper limit on the present cosmic mass density requires the Boltzmann factor $\exp(-m_L/kT_f)$ to be very small. For non-relativistic velocities, the cross-section $\sigma(v)$ for the exothermic processes (1) behaves like $1/v$, so we can take $\langle \sigma v \rangle$ as a temperature-independent constant. If there were only a single annihilation channel open (say $L\bar{L}^0 \rightarrow \nu\bar{\nu}$) and if the annihilation process was due to an ordinary "V minus A" charged-current Fermi interaction, then $\langle \sigma v \rangle$ would be just $G_F^2 m_L^2 / 2\pi$, where G_F is the Fermi coupling constant $1.15 \times 10^{-5} \text{GeV}^{-2}$. To take account of more general

possibilities, we shall write

$$\langle \sigma v \rangle = G_F^2 m_L^2 N_A / 2\pi \quad (6)$$

where N_A is a dimensionless fudge factor which depends both on the number of annihilation channels open and on the details of the weak $L\bar{L}^0$ annihilation interaction. (In general, the annihilation proceeds both through exchange of charged and of neutral intermediate vector bosons.) Where a numerical value is needed, we will take $N_A = 14$. [This corresponds to the annihilation channels $L\bar{L}^0 \rightarrow \nu_e \bar{\nu}_e, \nu_\mu \bar{\nu}_\mu, \nu_\tau \bar{\nu}_\tau, e^+ e^-, \mu^+ \mu^-, u\bar{u}, d\bar{d}, s\bar{s}$, with all fermions (antifermions) of helicity $-k$ ($+k$), and three colors for each of the quarks u, d, s .] New channels open up for m_L above about 5 GeV, leading to a moderate rise in N_A .

Equations (3)-(6) allow us to rewrite the rate equation (2) in the convenient form

$$df/dx = C\mu^3 (f^2 - f_0^2) \quad (7)$$

where

$$x = kT/m_L, \quad \mu^3 = m_L^3 N_A / \sqrt{N_F},$$

$$f(x; \mu) = n/T^3, \tag{8}$$

$$f_0(x) = n_0/T^3 = \pi^{-2} k^3 \int_0^\infty du u^2 \{ \exp(x^{-2} + u^2)^{-1/2} + 1 \}^{-1},$$

and

$$C = \frac{1}{k} G_F^2 \left(\frac{45}{32\pi^5 G} \right)^{1/2} = 1.28 \times 10^6 \left(\frac{\text{cm}^2 \text{K}}{\text{GeV}} \right)^{1/2}. \tag{9}$$

Equation (7) is to be solved subject to the initial condition that as $x \rightarrow \infty$, $f(x)$ approaches the equilibrium value $f_0(x)$. For $x \ll 1$, $f(x)$ approaches an asymptotic value, which evidently depends only on the single unknown parameter μ :

$$\lim_{x \rightarrow 0} f(x; \mu) = F(\mu) \tag{10}$$

The subsequent annihilation of electron-positron pairs increases kT by a factor $(11/4)^{1/3}$, and hence decreases n/T^3 by a factor $4/11$, so the present mass density of heavy neutrinos (and antineutrinos) will be given by

$$\rho_L = 2 \left(\frac{4}{11} \right)^{1/3} m_L T_Y^3 F(\mu) \tag{11}$$

with $T_Y = 3 \text{K}$ the present radiation temperature. (For freezing temperature above 100 MeV the factor $4/11$ must be decreased to take account of the subsequent heating of photons by annihilation of $\mu^+ \mu^-$, $\pi^+ \pi^-$, etc.).

Computer solutions of Eq. (7) are shown for a variety of special cases in Fig. 1. We find that over the range of parameters considered here, the function F of Eq. (10) can be very well represented by

$$F(\mu) = 1.20 \times 10^{-5} (\text{cm}^2 \text{K})^{-3} \{ \mu (\text{GeV}) \}^{-2.85} \tag{12}$$

The present mass density of heavy neutrinos and antineutrinos is then given by Eq. (11) as

$$\rho_L = (4.2 \times 10^{-28} \text{g/cm}^3) \times (m_L (\text{GeV}))^{-1.85} (N_A / \sqrt{N_F})^{0.95}. \tag{13}$$

If we require that $\rho_L < 2 \times 10^{-29} \text{g/cm}^3$, then m_L must be subject to the lower bound

$$m_L (N_A / \sqrt{N_F})^{0.51} \geq 5.2 \text{ GeV}. \tag{14}$$

For $N_A = 14$, $N_F = 4.5$, this gives $m_L \geq 2 \text{ GeV}$. Allowing for a factor of 4 uncertainty in the $L_{\nu 0}$ annihilation rate, the lower bound on m_L would lie in the range 1-4 GeV.

It is enlightening to see how these results can be obtained by an analytic approximation to the asymptotic value of n/T^3 .

We expect that $f(x)$ remains approximately equal to $f_0(x)$ until the temperature drops to a freezing value T_f , at which the annihilation rate per unit volume equals the rate of change of n_0 , or

$$df_0/dx = C \mu^3 f_0^2, \quad \text{at } x = x_f = kT_f/m_L, \tag{15}$$

and that thereafter f obeys approximately the equation

$$df/dx = C \mu^3 f^2, \quad x < x_f, \tag{16}$$

with the initial condition $f(x_f) = f_0(x_f)$. Since $kT_f \ll m_L$, we can approximate f_0 in Eq. (8) by the non-relativistic formula

$$f_0(x) = \frac{n_0}{T^3} = 2K^3 (2\pi x)^{-3/2} e^{-1/x} \tag{17}$$

and take only the exponential into account in evaluating df_0/dx . The freezing temperature is then obtained from

REFERENCES

- ¹See e.g., G. Marx and A.S. Szalay, Proceeding of the Neutrino-1972 Conference, A. Frankel and G. Marx, eds. (OMDK-TECHNIFORM, Budapest, 1972), Vol. I, p. 191; R. Cowsik and J. McClelland, Phys. Rev. Lett. **29**, 669 (1972), and Ap. J. **180**, 7 (1973).
- ²For a review with references to the original literature, see S. Weinberg, Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity, (Wiley, New York, 1972), Chapter 15.

- ³This includes a factor 3/4 for the effects of Fermi statistics; a factor 4/11 because the neutrino temperature is $(4/11)^{1/3}$ the photon temperature; and a factor of 2 because both neutrinos and antineutrinos are included. The massive neutrinos are, like the photons, supposed to exist in two helicity states.
- ⁴To take account of the uncertainty in galactic evolution, we assume that $q_0 < 2$. A value $H_0 = 50.3 \pm 4.3$ km/sec/Mpc is given by A. Sandage and G.A. Tammann, Ap.J. **210**, 7 (1976). The density 2×10^{-29} g/cm³ corresponds to $q_0 = 2$ and $H_0 = 50$ km/sec/Mpc.
- ⁵See e.g., B.W.Lee and S. Weinberg, FERMILAB-Pub-77/31-THY, to be published. It is possible that none of the heavy neutral leptons in this model are absolutely stable, in which case the lower bound on m_L derived here need not be satisfied.

$$x_f^{-1/2} e^{-1/2} = C k^3 (2\pi)^{-3/2} \mu^3 \quad (18)$$

$$= 1.35 \times 10^7 N_A N_F^{-1/2} [m_L (\text{GeV})]^3$$

For μ in the range 1 to 10 GeV, we find x_f^{-1} between 15 to 22. Thus, from Eqs. (17)-(18), we find that

$$f(x_f) = f_0(x_f) = (C\mu x_f^2)^{-1} \quad (19)$$

Equation (16) can now be solved:

$$f(x) = \{f_0(x_f)^{-1} + C\mu^3(x_f - x)^{-1}\}^{-1} \quad (20)$$

so that, with $x_f^{-1} = 20$,

$$F = f(0) = (C\mu^3 x_f^2)^{-1} (1 + x_f^{-1})^{-1} \\ = 1.5 \times 10^{-5} [\mu (\text{GeV})]^{-3} (\text{cm}^3 \text{K})^{-3} \quad (21)$$

in reasonable agreement with the more accurate result (12).

This analysis shows that the freezing temperature is likely to be about 1/20 the heavy neutrino mass, or 100 MeV for $m_L = 2$ GeV.

Of course, if a stable heavy neutral lepton were discovered with a mass of order 1-15 GeV, the gravitational field of these heavy neutrinos would provide a plausible mechanism for closing the universe.

One of us (S.W.) would like to thank Robert Wagoner for helpful conversations, and the other (B.W.L.) wishes to thank Robert Pearson for his assistance in computer calculations.

6 We suppose here that the chemical potentials associated with any conserved quantum numbers carried by L^0 are vanishingly small. If there were any appreciable degeneracy, the mass density of the heavy neutrinos would greatly exceed reasonable cosmological limits.

7 This includes contributions of $\gamma, \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, e^+, e^-, \mu^+, \mu^-$; plus three color triplets u, d, s of quarks and the corresponding antiquarks; plus eight massless spin-one gluons.

FIGURE CAPTION

Fig. 1 n/T^3 vs. T for a variety of special cases of m_L, N_F and N_A .

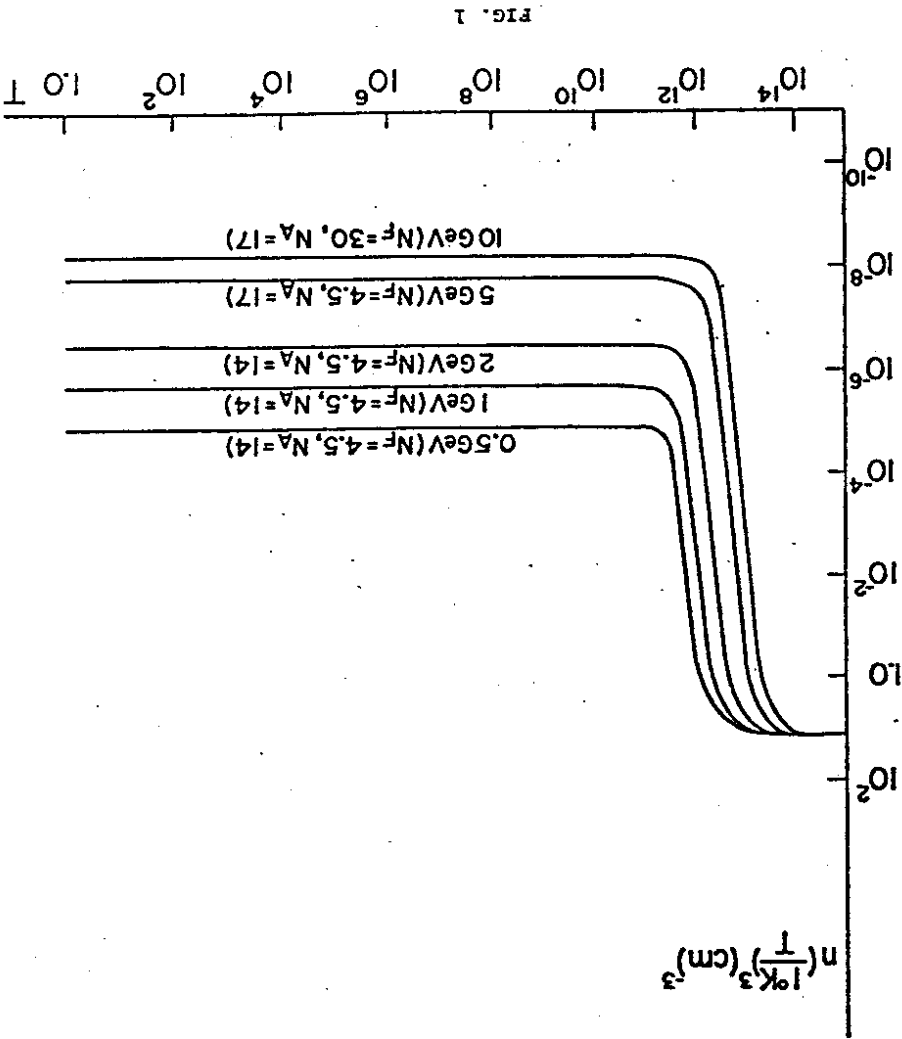


FIG. 1