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LIMIT ON THE REST MASSES FROM BIG BANG COSMOLOGY

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The age of the Universe and the Hubble deceleration of galaxies depends upon the average mass density. The temperature of the electromagnetic background radiation determines also the neutrino particle density. These empirical informations put an upper limit on the rest masses of the neutrinos, which are more restrictive than the laboratory values.

§ 1. Introduction

Our direct experimental information about the neutrino rest masses is rather poor:

$$m_{\nu_e} < 60\,\text{eV}\,[1], \qquad m_{\nu_\mu} < 1.6\,\text{MeV}\,[2]\,. \tag{1}$$

There is, however, a possibility to obtain more restrictive upper limits on these masses from the empirical cosmology. The Universe is filled by a 2.7 °K black body radiation, discovered by PENZIAS and WILSON in 1965 [3]. To detect the corresponding neutrino background, produced at the time of the Big Bang, would be a much harder job, since the direct neutrino interactions are weak ones. On the other hand, the neutrinos may carry a considerable part of the overall mass density of the Universe, consequently their gravity might play an important role in the evolution of the Universe.

In the early stage of this evolution a high number of particle-antiparticle pairs were in thermal equilibrium with the radiation field. As the temperature decreased, all the particle pairs annihilated but the neutrinos. They found themselves decoupled from the charged particles and photons, thus their number did not change any more. The number of neutrinos must be enormous even to-day as compared to the number of the photons and much higher than the number of the charged atomic constituents. During the expansion of the Universe the behaviour of massive and massless particles is quite different. (In case of massive particles — e.g. atoms — the particle number is fixed, the volume increases as R^3, consequently the particle density goes with R^{-3} and so does the energy density, too. In the case of massless particles — e.g. photons — the wavelength is proportional to R, the energy to R^{-3}, consequently the energy density goes with R^{-4}.) Even a very tiny neutrino rest mass — if there is any — might have a vast influence in shaping the face of our present world. This offers us a way to learn the value of the neutrino rest mass from actual astronomical observations.

The aim of the present paper is to investigate the influence of m_{ν_e} and m_{ν_μ} on the evolution of the Universe and to put an upper limit on them from the expansion deceleration q_0 and from the age t_0 of the Universe.

§ 2. Geometry of the Universe

An isotopic homogeneous model of the Universe will be assumed, which may be described by the Robertson—Walker metric:

$$ds^2 = c^2dt^2 - R(t)^2 \frac{dr^2 + r^2d\vartheta^2 + \sin^2\vartheta dp^2}{(1 + kr^2/4)^2} . \tag{2}$$

The dimensionless parameter k is characteristic for the qualitative space-time structure of the Universe:

$k = -1$ means an open space with hyperbolic geometry and with infinite volume;

$k = 0$ means a flat Euclidian space with infinite volume,

$k = +1$ means a closed space with spherical geometry and with finite volume.

As the scale factor $R(t)$ varies with time, all the lengths remain proportional to it. (This produces the red shift of light and the recession of galaxies.) The time dependence of $R(t)$ is described by the Einstein equations, which take the following from for the metric (2):

$$\left(\frac{\dot{R}}{R}\right) + k\frac{c^2}{R^2} = \frac{8\pi G}{3}\varrho , \tag{3}$$

$$\frac{d}{dt}(\varrho c^2R^3) + P\frac{d}{dt}(R^3) = 0 . \tag{4}$$

Here ϱ means the mass density and P the pressure of matter. These two equations lead to a singularity $R = 0$. (It is convenient to choose $t = 0$ at the singular moment.) During the evolution of the Universe the time dependence of $R(t)$ may have three different forms (Fig. 1):

If $k = -1$, the expansion slows down all the time, but without stopping.

If $k = 0$, the expansion slows down and stops at $t = +\infty$

If $k = -1$, the expansion slows down, stops, then turns over to contraction.

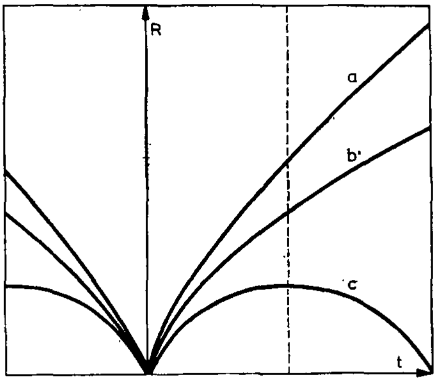

Fig. 1. The time-dependence of $R(t)$ in terms of the age of the Universe. The curves a, b, c correspond to the values of $k = -1, 0, 1$. (Both scales are nonlinear.)

The present state ($t = t_0$) of the expansion is usually characterized by two observable quantities: by the Hubble parameter H_0 and by the deceleration parameter q_0 [4]:

$$H_0 = \frac{\dot{R}(t_0)}{R(t_0)} = 53 \pm 5 \text{ km/s Mpc} = \left[(18.4 \pm 2) \cdot 10^9 \text{ years}\right]^{-1}, \tag{5}$$

$$q_0 = -\frac{\ddot{R}(t_0)R(t_0)}{\dot{R}(t_0)^2} = 0.94 \pm 0{,}4, \tag{6}$$

k, H_0 and q_0 are related to each other by the equation

$$k\frac{c^2}{R^2} = H_0^2(2q_0 - 1) - \frac{8\pi G}{3}p, \qquad p = 0.$$

This shows, that $q_0 > 0.5$ means $k = +1$, $q_0 < 0.5$ means $k = -1$. The observed value (6) suggests evidently a closed Universe with $k = +1$. As it can be seen from the relation

$$\varrho = \frac{3}{8\pi G}\left(k\frac{c^2}{R^2} + H_0^2\right) = \frac{3}{4\pi G}H_0^2 q_0$$

the mean values $H_0 = 53$ km/sMpc and $q_0 = 0.94$ give a mass density $\varrho(t_0) = 10^{-29}$ g/cm^3 for our present Universe, which is considerably higher than the optically observed stellar mass density $\varrho_* = 0.03 \cdot 10^{-29}$ g/cm^3. (The latter value alone would give an open universe with $k = -1$.) These numbers show that a considerable neutrino contribution cannot be ruled out by the present astronomical evidence.

§ 3. Thermodynamics in the Lepton Era

After the annihilation of the hadrons the matter was present in the form of photons, leptons and nucleons. The number of the surviving nucleons was determined by the conservation of the baryonic charge, the number of leptons and photons by the temperature. This has the consequence, that in a certain period of the cosmological evolution, in the temperature range $m_\pi c^2 > kT > m_e c^2$ the most abundant particles were the leptons. In this Lepton Era the density was still so high that collisions were rather frequent. All the particles

$$\gamma, e^+, e^-, \mu^+, \mu^-, \nu_e, \tilde{\nu}_e, \nu_\mu, \tilde{\nu}_\mu$$

were in thermal equilibrium, their energy densities were nearly equal. (The only differences were caused by the spin degrees of freedom and the different statistics.) If one makes the assumption, more pessimistic from the point of view of the neutrinos that the right-handed neutrino states and left-handed antineutrino states, appearing as a consequence of $m_\nu \neq 0$, have not had time to be filled up, one gets

$$\varrho_{e^+e^-} = \varrho_{\mu^+\mu^-} = \frac{7}{4}\varrho_\gamma , \qquad \varrho_{\nu_e\bar{\nu}_e} = \varrho_{\nu_\mu\bar{\nu}_\mu} = \frac{7}{8}\varrho_\gamma . \tag{7}$$

The leptons are coupled together by the electromagnetic and weak interactions:

$$e^+ + e^- \rightleftharpoons 2\gamma \rightleftharpoons \mu^+ + \mu^- ,$$

$$e^+ + e^- \rightleftharpoons \nu_e + \tilde{\nu}_e , \qquad \mu^+ + \mu^- \rightleftharpoons \nu_\mu + \tilde{\nu}_\mu , \qquad e^- + \mu^+ \rightleftharpoons \nu_e + \tilde{\nu}_\mu, \text{ etc.} \tag{8}$$

As the temperature drops below the value $kT = m_\mu c^2$, the muons start to disappear, their energy and entropy flows over into the lighter particles. The μ-neutrinos do not collide with muons any more, they do not have energy enough to produce newer muons. They become decoupled from the rest of the particles. (To be more precise, knowing the cross sections from the theory of weak interactions, one can calculate the average time between two successive interactions. One can speak about decoupling, when this time becomes longer than the lifetime of the Universe. The average interaction time depends upon the temperature very sensitively. The detailed calculation of the decoupling temperature will be given in the Appendix.)

The decoupling temperature of the electron-neutrinos can be obtained on the same line. So the four characteristic temperatures of the Lepton Era turn out numerically as follows:

$$\text{annihilation of } \mu^+\mu^- \text{ at } T_\mu = 120 \cdot 10^{10}\,{}^\circ\text{K}, \tag{9}$$

$$\text{decoupling of } \nu_\mu\nu_\mu \text{ at } T_{\nu_\mu} = 12 \cdot 10^{10}\,{}^\circ\text{K}, \tag{10}$$

$$\text{decoupling of } \nu_e\nu_e \text{ at } T_{\nu_e} = 1.8 \cdot 10^{10}\,{}^\circ\text{K}, \tag{11}$$

$$\text{annihilation of } e^+e^- \text{ at } T_e = 0.59 \cdot 10^{10}\,{}^\circ\text{K}. \tag{12}$$

The number of neutrinos froze in at decoupling, the energy of an individual particle decreases as a consequence of the expansion of space. The entropy and energy of muons and electron pairs flows into that of the photons during the annihilation periods. The entropy flow is shown by Fig. 2.

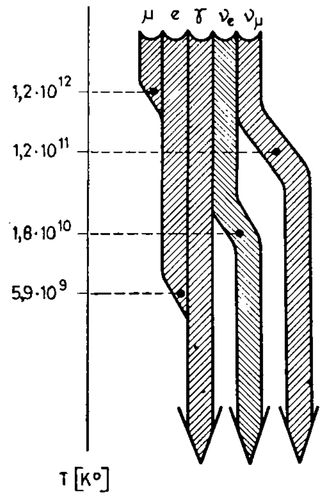

Fig. 2. The entropy flow in the Universe between the different particles. When different channels touch each other, they are in equilibrium

§ 4. Calculation of the density and pressure functions

We are going to start the calculation after the annihilation of the muons, at the decoupling temperature of the μ-neutrinos (T_{ν_μ}). At this temperature the μ-neutrinos are still in equilibrium with the other components. From this point it is easy to follow the life story of all the particles exactly.

Photons: The photon distribution is given by Planck's law. The energy density and pressure are given by the Stefan—Boltzmann-formula:

$$\varrho_\gamma = aT^4, \qquad P_\gamma = \frac{1}{3}\varrho_\gamma, \qquad a = 7.569 \cdot 10^{-15}\,\mathrm{erg/cm^3\,{}^\circ K^4}. \tag{13}$$

The wave-length is changing proportionally to the scale factor R, consequently according to Wien's displacement law the temperature depends on R on a very simple way:

$$RT = \text{const.} \tag{14}$$

Electrons: They obey the Fermi statistics. Since in the Lepton Era the abundancy of electron pairs was much higher than that of the protons, the chemical potential may be taken zero. At high temperatures one must use relativistic energies, $\varepsilon = (p^2 + m^2)^{1/2}$. Consequently,

$$\varrho_{e^+e^-} = \frac{16\pi}{k^3}\int_0^\infty \varepsilon(p)\frac{p^2dp}{1+e^{\varepsilon/kT}}\,.$$

With the new variables x and θ, defined by the relations

$$x = m_ec^2/kT, \qquad \varepsilon = x\cosh\theta, \qquad p = x\sinh\theta\,, \tag{15}$$

the integral can be given in a form more convenient for numerical computations [4]:

$$\varrho_{e^+e^-} = 16\pi\left(\frac{m_ec}{h}\right)^2 m_ec^2\int_0^\infty \frac{\sinh^2\theta\cosh^2\theta}{1+\exp(x\cosh\theta)}\,d\theta\,,$$

$$p_{e^+e^-} = \frac{16\pi}{3}\left(\frac{m_ec}{h}\right)^3 m_ec^2\int_0^\infty \frac{\sinh^4\theta}{1+\exp(x\cosh\theta)}\,d\theta\,.$$

Let us define the following functions:

$$f_0 = \int_0^\infty \frac{d\theta}{1+\exp(x\cosh\theta)} = \sum_{n=1}^\infty(-1)^{n+1}K_0(nx)\,, \tag{16}$$

$$f_1 = \int_0^\infty \frac{\sinh^2\theta\,d\theta}{1+\exp(x\cosh\theta)}\sum_{n=1}^\infty(-1)^{n+1}\frac{K_1(nx)}{nx}\,, \tag{17}$$

$$f_2 = \int_0^\infty \frac{\sinh^4\theta d\theta}{1+\exp(x\cosh\theta)}\sum_{n=1}^\infty(-1)^{n+1}\frac{K_2(nx)}{(nx)^2}\,. \tag{18}$$

(K_r is the modified Bessel function of rank r.) So one has obtained a very rapidly converging series expansion. In order to reach an accuracy of 10^{-4} it is enough to sum up six terms. The density and pressure functions can be evaluated in terms of f_0, f_1, f_2:

$$\varrho_{e^+e^-} = 2a_e(f_1 + f_2)\,, \tag{19}$$

$$p_{e^+e^-} = \frac{2a_e}{3}f_2\,, \tag{20}$$

$$a_e = 8\pi\left(\frac{mc}{h}\right)^3 mc^2 = 1.44\cdot 10^{24}\,\mathrm{erg/cm^3}\,.$$

At high temperatures ($\varepsilon T \gg m_ec^2$) the electrons behave like radiation, i.e.

$$\varrho_{e^+e^-} = \frac{7}{4}\varrho_\gamma\,, \qquad p_{e^+e^-} = \frac{1}{3}\varrho_{e^+e^-}\,. \tag{21}$$

In the neighbourhood of $kT = m_ec^2$ the energy density and pressure drops very fast, the energy and entropy flows over into the photon component. $\varrho_{e^+e^-}/\varrho_\gamma$ and $p_{e^+e^-}/p_\gamma$ are shown in terms of the temperature on Fig. 3.

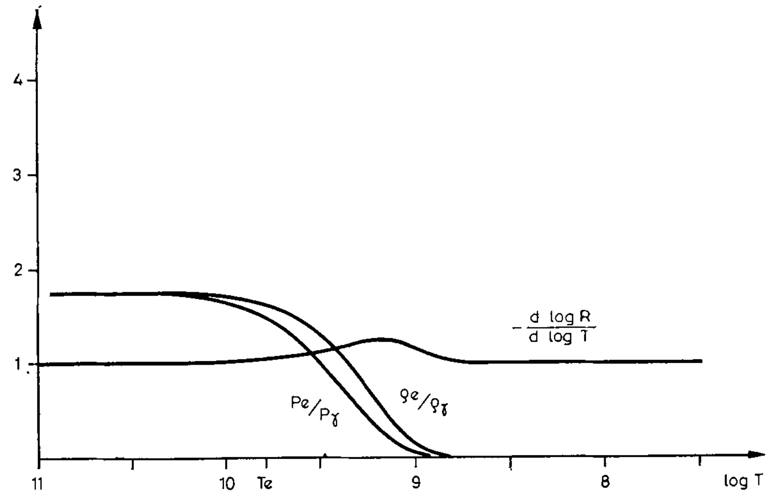

Fig. 3. The temperature-dependence of $Pe^{+}e^{-}/P\gamma$, $\varrho e^{+}e^{-}/\varrho\gamma$ and $-d\ln R/d\ln T$

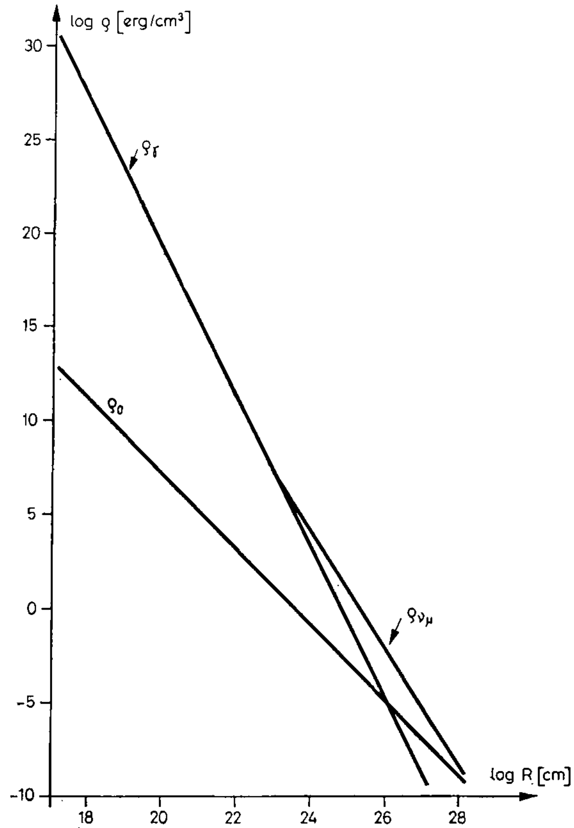

Fig. 4. The R-dependence of the different densities. The breaking point corresponds to $mv_{\mu} = 50$ eV, approximately. The crossing point of the curves ϱ and ϱ_0 determines the end of the expansion of the Universe

Electron-neutrinos: In the case of vanishing rest mass one has thermal distribution before decoupling:

$$\varrho_{\nu_e} = \frac{7}{8} aT^4, \qquad p_{\nu_e} = \frac{1}{3} \varrho_{\nu_e}, \quad \text{if} \quad T > T_{\nu_e}. \tag{22}$$

In the moment of decoupling the temperature is T_{ν_e}, the scale factor is R_{ν_e}. From this point on each neutrino looses energy via the Hubble shift, according to the law $p \sim \lambda^{-1} \sim R^{-1}$. The R dependence of ϱ and p is radiation-type:

$$\varrho_{\nu_e} = \frac{7}{8} aT_{\nu_e}^4 \cdot \left(\frac{R_{\nu_e}}{R}\right)^4 \qquad p_{\nu_e} = \frac{1}{3} \varrho_{\nu_e} \quad \text{if} \quad T < T_{\nu_e}$$

One sees that the formulas (22) can be used in the whole region.

Muon-neutrinos: We are interested in the possibility that μ-neutrinos may have a nonvanishing rest mass, and this makes life harder. At the moment of the decoupling $(T_{\nu_\mu}, R_{\nu_\mu})$ there is still an equilibrium distribution:

$$dn(p) = \frac{8\pi}{h^3} \frac{p^2 dp}{1 + \exp[(p^2 + m_{\nu_\mu}^2)^{1/2}/kT_{\nu_\mu}]}. \tag{23}$$

After having been decoupled, the number of neutrinos does not change any longer. The momentum of each individual particle will decrease proportionally to R^{-L},

$$p^2 = p\left(\frac{R_{\nu_\mu}}{R}\right),$$

consequently the total energy density may be obtained by integrating $\varepsilon' = (p^2 + m_{\nu_\mu}^2)^{1/2}$ over the distribution (23):

$$\varrho_{\nu_\mu} = \frac{8\pi}{h^3} \int_0^\infty \left[m_{\nu_\mu}^2 + p^2\left(\frac{R_{\nu_\mu}}{R}\right)^2\right] \cdot \frac{p^2 dp}{1 + \exp\left[(p^2 + m_{\nu_\mu}^2)^{1/2}/kT_{\nu_\mu}\right]}.$$

The pressure can be computed from the formula as $p = -dE/dV$. Using the same substitutions as in the case of electrons, writing

$$\varepsilon(R) = \left[1 + \left(\frac{R_{\nu_\mu}}{R}\right)^2 \sinh^2\theta\right]^{1/2}$$

one gets the energy density and pressure:

$$\varrho_{\nu_\mu\nu_\mu} = \left(\frac{R_{\nu_\mu}}{R}\right)^3 \left(\frac{m_{\nu_\mu}}{m_e}\right)^4 a_e \int_0^\infty \varepsilon(R) \frac{\sinh^2\theta \cosh\theta d\theta}{1 + \exp(x\cosh\theta)}, \tag{24}$$

$$p_{\nu_\mu\nu_\mu} = \left(\frac{R_{\nu_\mu}}{R}\right)^5 \left(\frac{M_{\nu_\mu}}{M_e}\right)^4 \frac{a_e}{3} \int_0^\infty \frac{\cosh\theta}{\varepsilon(R)} \frac{\sinh^4\theta d\theta}{1 + \exp(x\cosh\theta)}. \tag{25}$$

These expressions can be integrated numerically by conventional methods. (No series expansion is allowed, because the value of R changes 11 orders of magnitude.)

If one supposes a nonvanishing rest mass also for the electron neutrinos, they must be treated in the same way as the muon neutrinos.

If the right-handed neutrino states and left-handed antineutrino states have had time enough to be filled up, the e-neutrino and μ-neutrino density and pressure values must be doubled.

Knowing the energy density and pressure in terms of R, one can start to integrate the Einstein equations (3), (4), which give us the history of the expansion.

§ 5. Integration of the Einstein equations

If $\varrho(R)$ is known from § 4, one can integrate the equation (3), if the initial conditions are known. Let us start the integration at the moment, when T cools down to T_{ν_μ} and the μ-neutrinos become decoupled. Since this is well within 1 second after the Big Bang, we may put $t = 0$. Let us choose $R(o)$ arbitrarily. (The neutrino rest masses m_{ν_e} and m_{ν_μ} are other free parameters in the calculation.)

The computer calculation of $R(t)$ is made to stop, when the photon temperature $T_\gamma(t)$ reaches its present value 2.7 °K. The corresponding $t = t_0$ is the age of the Universe. From the $R(t)$ function one can obtain the actual values of H_0 and q_0. In this way to every choice of $R(o)$, m_{ν_e} and m_{ν_μ} one obtains definite values for t_0, H_0 and q_0. If the latter numbers are known empirically, one can find the correct values of $R(o)$, m_{ν_e} and m_{ν_μ}. In this way the neutrino rest masses are available from real astronomical observations.

In the actual computation is advantageous to separate the photon-electron-component ϱ_1 from the neutrino component ϱ_2. The former are always sticked together by the electromagnetic interaction.

From the Einstein equations (3), (4) it follows

$$\frac{d\varrho}{dt} = -3\left(\frac{\dot{R}}{R}\right)(p+\varrho).$$

The left-hand side can be written as follows:

$$\frac{d\varrho}{dt} = \frac{d\varrho}{d\ln T}\cdot\frac{d\ln T}{dt} + \frac{d\varrho}{d\ln R}\cdot\frac{d\ln R}{dt}.$$

After some simple tricks one arrives to the relation

$$\frac{d\varrho_1}{d\ln T} = -3(p_1+\varrho_1)\frac{d\ln R}{d\ln T}. \tag{26}$$

By making use of the results of § 4, one can write this in the following form:

$$\frac{d \ln R}{d \ln T} = -1 - \frac{2a_e}{3} \cdot \frac{f_0 + 2f_1}{p_1 + \varrho_1} .$$

Since in the adiabatic equation of state of the radiation field $d \ln R / d \ln T = -1$, the first term of the right-hand side describes the adiabatic cooling of radiation, caused by the expansion, the second term describes the influence of pair annihilation on the radiation. This entropy transition is important only around the temperature T_e (Fig. 3).

By taking this remark into account, Eq. (26) can be integrated after some simple substitutions. The integration constant can be obtained by considering that the electron in high temperature limit must behave radiation-like.

$$R = \frac{R_{\nu_\mu} T_{\nu_\mu}}{T} \left[\frac{11/3aT^4}{p_1 + \varrho_1} \right]^{1/3} . \tag{27}$$

This gives the scale factor R at the moment when the temperature of the universe is T.

The only open question left is the time dependence of the expansion. This is described by the Einstein equation (3). For the actual work it is convenient to eliminate R from this differential equation with the help of the formula (27), so one gets a differential equation for the function

$$\frac{d \ln T}{d \ln t} = \frac{d \ln T}{d \ln R} \cdot \frac{d \ln R}{d \ln t} = -t \frac{\left[\frac{8\pi G}{3} \varrho - \frac{c^2}{R^2} \right]^{1/2}}{1 + \frac{2a_e}{3} \frac{f_0 + 2f_1}{p_1 + \varrho_1}} . \tag{28}$$

This equation is integrated by computer.

The density runs proportional to R^{-4} first (radiation-type Universe), as is shown in Fig. 5. At the temperature $kT = m_{\nu_\mu} c^2$ there is a breaking point on the density curve: the slope changes, because the density starts to behave like R^{-3} (rest mass-type Universe). If m_{ν_μ} is larger than 3—4 eV, this change happens before the protons became dominating. In this case the presence of the atoms may be neglected in discussing the cosmological evolution of the Universe. The time corresponding to the breaking point which is important for the character of the expansion, depends sensitively on m_{ν_μ}. This is the explanation, why the observable astronomical values H_0, q_0, t_0 depend on the neutrino mass so sensitively.

§ 6. Conclusion

In a previous calculation Gershtein and Zel'dovich obtained the estimation

$$m_{\nu_\mu} < 200 \text{ eV}$$

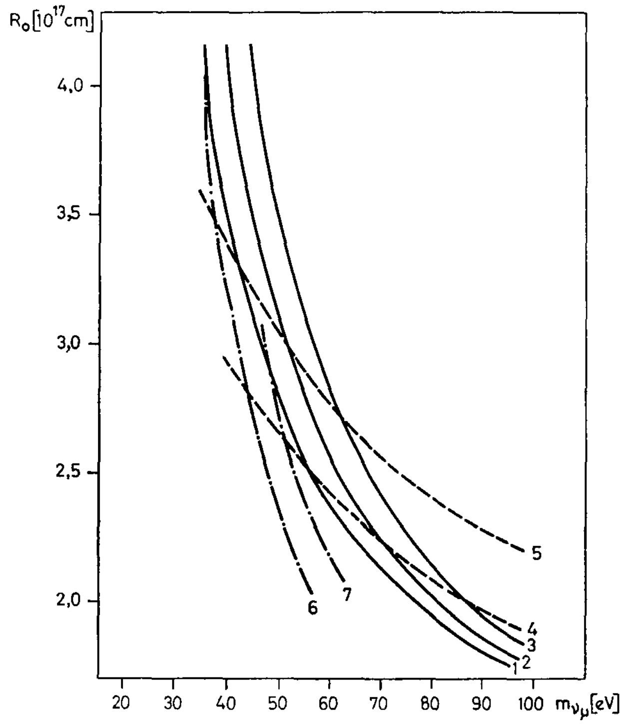

Fig. 5. The graph of H_0, q_0 and t_0 in terms of $m\nu_\mu$ and R_0, the early radius of the Universe. The solid curves 1, 2, 3 correspond to the values $H_0 = 48$, 53, 58 km/s.Mpc. The dashed lines 4, 5 correspond to $q_0 = 1.34$ and 0.94. The dotted-dashed lines 6, 7 correspond to the values $t_0 = 12$ and 11 Gyears, respectively

from the age of the moon rocks ($t_0 < 4.5 \cdot 10^9$ years) in their pioneering work [6]. The present authors made use of the observed Hubble parameter (5) and from the conservative deceleration limit $q_0 < 2$ to deduce a sharper limit on the neutrino mass m_{ν_μ} [7]:

$$m_{\nu_\mu} < 140 \text{ eV}. \tag{29}$$

Later Cowsik and McClelland used the optimistic value $q_0 = 0.94$ and they concluded [8] in

$$\Sigma m_\nu < 66 \text{ eV}. \tag{30}$$

In the present fluid state of the empirical cosmology it may be a better tactics to leave the choice from observational data to the reader. We summarize the results of our calculations in Figs 5, 6. Here the values of t_0, H_0 and q_0 have been plotted in terms of m_{ν_μ} and $R(0)$. A more informative diagram is shown in Fig. 7, where t_0 and q_0 are plotted for different possible values of H_0 and m_{ν_μ}.

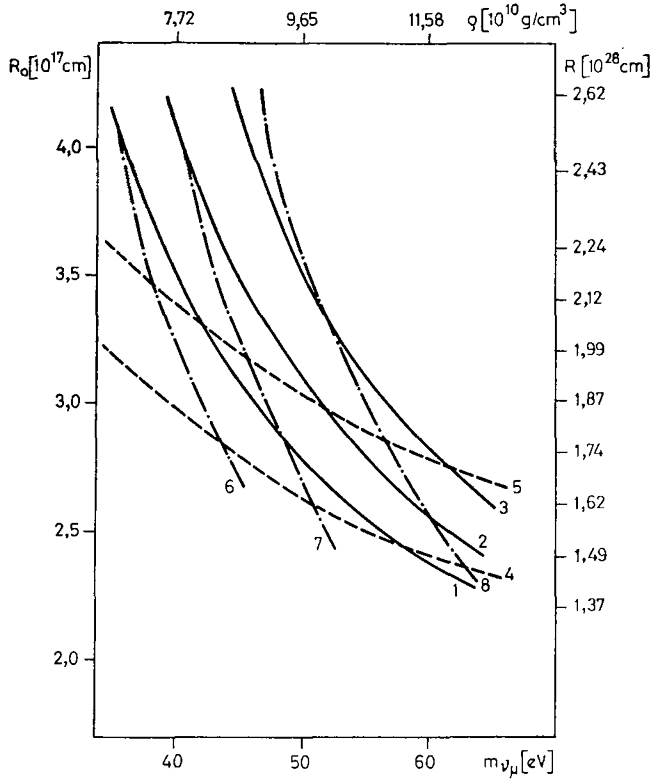

Fig. 6. Same as Fig. 5, only the range of $m\nu_\mu$ is different. The crossing points, giving the limits on the ν_μ rest mass can be seen on this figure. The scale on the right hand side shows the values of R (the radius of the Universe now). The units are 10^{28} cm-s. On the upper scale the energy density of the Universe is given. The solid curves 1, 2, 3 correspond to $H_0 = 48$, 53 and 58 km/s.Mpc, the dashed lines 4,5 correspond to $q_0 = 1.34$ and 0.94, the dotted-dashed lines correspond to $t_0 = 12, 11, 10$ Gy

In these calculations the most pessimistic assumptions were used: The evolution of the Universe was too fast to fill up the right-handed ν and left-handed $\bar{\nu}$ states, the ν_e rest mass equals zero, so the whole cosmological effect is carried by the ν_μ rest mass. If one drops these assumptions, on the horizontal axis in Figs 5, 6, 7, one can write the sum of the neutrino rest masses, summed over all the neutrino degrees of freedom, i.e. $\Sigma m_{\nu_\mu} = 4m_{\nu_e} + 4m_{\nu_\mu}$.

It is shown in these Figures, that the conclusion

$$m_{\nu_\mu} \leq \Sigma m_\nu < 90 \text{ eV} \tag{31}$$

is convincing even under moderate use of the data borrowed from the empirical cosmology. If we accept the nuclear age of our Galaxy [9]

$$t_G = (12 \pm 2) \cdot 10^9 \text{ years}, \tag{32}$$

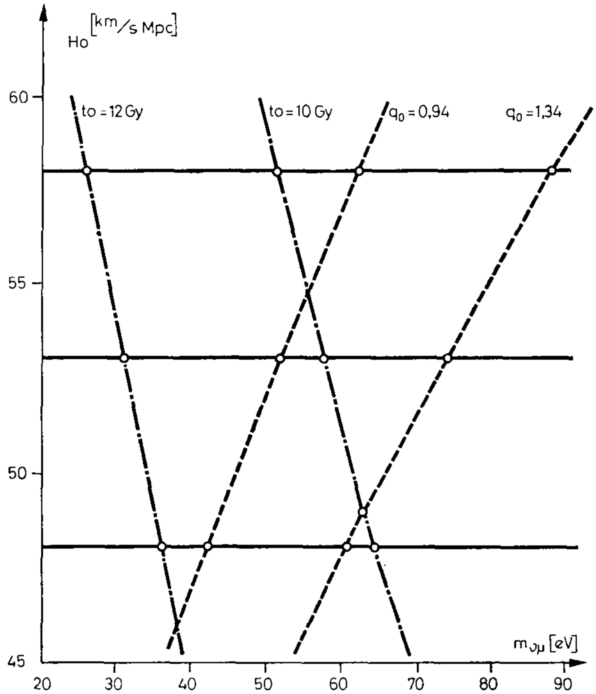

Fig. 7. Same as Figs. 5 and 6, only the R_0 dependence is eliminated. The parameters q_0 and t_0 are plotted in terms of $m_{\nu\mu}$ and H_0. The possible values of the parameters are laying within the area, bounded by the lines $H_0 = 48$, $H_0 = 58$ (km/s.Mpc), $q_0 = 1.34$ and $t_0 = 10$ Gy

one can put a more restrictive upper limit

$$m_{\nu_\mu} \leq \Sigma m_\nu < 64 \text{ eV}. \tag{33}$$

If one had $m_{\nu_e} = m_{\nu_\mu}$ and if all the eight ν states were filled up, one would arrive at the optimistic conclusion

$$m_\nu < 22 \text{ eV} \quad \text{(or 16 eV, respectively)}.$$

These values are by four or five orders of magnitude more accurate, than the laboratory limit (1). (The inequalities might turn into equalities, if we were able to know definitely that neutrinos are the only dominating form of matter and if the astronomical data were free from any systematic error. In this case one could say that the dominating form of matter are neutrinos, practically at rest.)

Another, even more daring idea to measure the neutrino rest mass by watching the sky is based on the "missing mass" phenomenon in big clusters of galaxies, especially in the Coma cluster. This invisible mass, which stabilizes the cluster, may be interpreted as a neutrino concentration produced by the gravitational pull of the cluster [10]. The value of the neutrino rest mass estimated in this way turns out to be comparable — or even better — than that given in this paper. The idea of a tiny nonvanishing neutrino rest mass, suggested by astronomy, deserves further investigations.

Appendix

Calculation of the decoupling temperature

The abundancy of the neutrino-lepton interactions can be characterized by the average interaction time. This quantity is by definition

$$\tau_{\nu l}(T) = \frac{\langle n_\nu \rangle}{\langle n_\nu n_l \sigma \mid v \mid \rangle} . \tag{34}$$

n_ν and n_l are the number densities of neutrinos and leptons, respectively. σ is the cross section, $| v |$ is the relative velocity. $\tau_{\nu l}$ is extremely sensitive to the actual value of the temperature, it increases rapidly as temperature decreases.

The decoupling temperature: is by definition the temperature, below which the neutrinos practically do not interact with other particles any more. This happens when the average interaction time becomes equal to the age of the Universe:

$$\tau_{\nu l}(T) = t(T). \tag{35}$$

Such a sharp decoupling is an idealisation of the process, of course, but it is a good approximation, since the interaction time increases with the temperature very rapidly.

The reaction $\nu_e + e \rightarrow e + \nu_e$ was studied by J. BAHCALL [11] and T. DE GRAAF [12], they concluded in the average interaction time

$$\tau_{\nu e} = 7 \cdot 10^5 \; T_9^{-5} \sec \tag{36}$$

(T_9 denotes the temperature in 10^9 °K units.)

The reaction $e^+ + e^- \rightleftarrows \nu_e + \tilde{\nu}_e$ was considered by H. Y. CHIU [13]. The average interaction time is

$$\tau_{\nu\nu} = 1.5 \cdot 10^6 \, T_9^{-5} \sec . \tag{37}$$

The relation between the temperature and the age of the Universe is

$$t = 10^2 \; T_9^{-2} \sec . \tag{38}$$

The decoupling temperature for the μ-neutrinos, obtained from the first ($\nu_e + e \rightarrow e + \nu_e$) reaction, according to equation (35) is

$$T_{\nu_e} = 1.8 \cdot 10^{10} \, ^\circ\mathrm{K}. \tag{39}$$

This is higher, than the annihilation temperature of the electrons,

$$T_e = 5.9 \cdot 10^9\ {}^\circ\mathrm{K}. \tag{40}$$

The similar reactions for the μ-neutrinos are

$$\nu_\mu + \mu \to \mu + \nu_\mu, \tag{41}$$
$$\mu^+ + \mu^- \rightleftarrows \nu_\mu + \bar{\nu}_\mu.$$

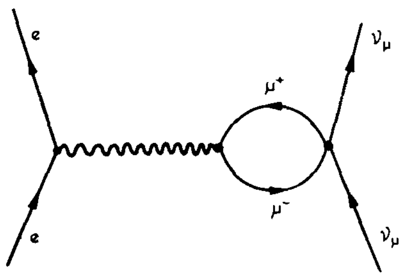

Fig. 8. The graph of the process $e + \nu_\mu \to \nu_\mu + e$

The average interaction time for the first reaction [12]

$$\tau_{\nu_\mu} = 7.5 T_9^{-7/2} \exp\left(\frac{1226}{T_9}\right) \sec. \tag{42}$$

The corresponding decoupling temperature is

$$T_{\nu_\mu} = 1.2 \cdot 10^{11}\ {}^\circ\mathrm{K}. \tag{43}$$

(The μ annihilation temperature is $T_\mu = 1.2 \cdot 10^{12}\ {}^\circ\mathrm{K}$.)

There is a possibility, however, that the reaction

$$\nu_\mu + e \to e + \nu_\mu$$

might play an important role, since below the μ annihilation temperature the number density of electrons is much higher, than the number density of muons. The corresponding graph is shown in Fig. 8; the μ-neutrino can be scattered by electrons through its electromagnetic form factor [14].

$$\langle \nu_\mu | J_{el} | \nu_\mu' \rangle = \frac{eG}{\sqrt{2}} \frac{1}{12\pi^2} F(q^2)^2 \frac{\bar{u}(\nu)}{(2\pi)^{3/2}} \{\gamma^\mu(1 - i\gamma^5) q^2 + 2Mq^\mu(i\gamma^5)\} \frac{u(\nu)}{(2\pi)^{3/2}},$$

where

$$F(q^2) = \ln \frac{\Lambda^2}{M^2} - \frac{5}{6} + \frac{11}{30} \frac{q^2}{M_\mu^2}, \qquad \Lambda = 300 \text{ Gev},$$

the cut-off parameter; $m_\mu = 106$ MeV, the muon rest mass; $q = p_\nu' - p_\nu$, the transferred momenta; M is the ν_μ rest mass. After some calculations one gets

$$\frac{d\sigma}{d\Omega} = \frac{e^4 G^2}{18} \frac{F(q^2)^2}{(2\pi^6)} \cdot \frac{1}{45} \{(S-\mu_e^2 - M^2)^2 + (S - \mu_e^2 - M^2 + t)^2 - 2m_e^2(2M - t)\}.$$

In order to determine the average interaction time, we have to integrate the following expression numerically:

$$\langle n_\nu n_e \sigma \mid v \mid \rangle = 8\pi^2 \int \mid v \mid \frac{d\sigma}{d\Omega} d\Omega f_\nu f_e dp_\nu dp_e d(\cos\theta), \tag{44}$$

where f_ν and f_e are the statistical distribution functions of the neutrinos and electrons. After the separation of some constant factors this expression can be written in the following form:

$$\langle n_\nu n_e \sigma \mid v \mid \rangle = 2.35 \cdot 10^{16} I(T). \tag{45}$$

Here $I(T)$ is a numerically calculable dimensionless function, depending on the temperature only.

The mean value of n_ν is

$$\langle n_\nu \rangle = 4.164 \cdot 10^{28} \; T_g^{3/2} \, \mathrm{cm}^{-3},$$

consequently

$$\tau_{\nu_e} = \frac{\langle n_\nu \rangle}{\langle n_\nu n_i \sigma \mid v \mid \rangle} = 7.769 \cdot 10^{12} \frac{T_g^{3/2}}{I(T)}. \tag{46}$$

After some numerical computation the decoupling temperature obtained from this reaction ($\nu_\mu + e \to e + \nu_\mu$) turns out to be $T_{\nu_\mu e} = 4.1 \cdot 10^{11}$ °K, which is higher, than the decoupling temperature of the $\nu_\mu + \mu \to \mu + \nu_\mu$ reaction (Fig. 9).

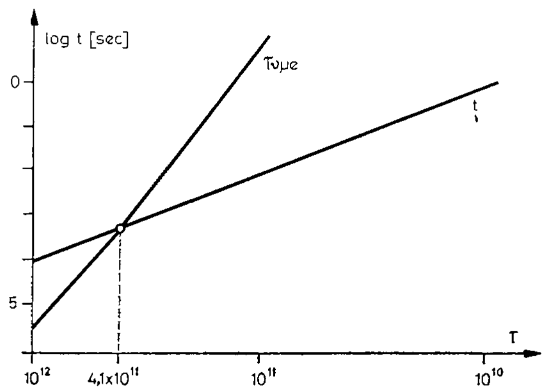

Fig. 9. Determination of the decoupling temperature from the average interaction time of the reaction $e + \nu_\mu \to \nu_\mu + e$. $t(T)$ is the age of the Universe, $\tau_{\nu\mu e}(T)$ the average interaction time, both depending on the temperature only. Their crossing point determines the decoupling temperature: $T_{\nu\mu e} = 4.1 \cdot 10^{11}$ °K

Another value of the decoupling temperatures can be calculated in Weinberg's model of weak and electromagnetic interactions. The $\nu_\mu + e \rightarrow e + \nu_\mu$ reaction has been considered by B. W. LEE et al. [15].

$$T(\nu_\mu + e \rightarrow e + \nu_\mu) = i \frac{G}{\sqrt{2}} \left(\frac{3\alpha}{4\pi}\right) \left(\frac{m_W}{53\ \text{GeV}}\right) [\nu\gamma^\alpha(1 - i\gamma^5)\nu]\,[e\gamma_\alpha(1 - i\gamma^5)e]\,.$$

By taking this into account, we get a decoupling temperature somewhat lower, about

$$T^W_{\nu_{\mu e}} = 3 \cdot 10^{11}\ {}^\circ\text{K}\,.$$

So we may conclude by saying that the decoupling of μ-neutrinos from the rest of particles takes place somewhere around the temperature

$$T_{\nu_\mu} = 1.2 \cdot 10^{11}\ {}^\circ\text{K}.$$

The uncertainty in the value of the decoupling temperature has a small influence on the expansion as a whole, since about the time of decoupling there are no large entropy exchanges between the other particles either, and on the other hand the decoupled neutrino rest masses are negligible, so the adiabatic cooling and the Hubble shift of the individual particles produce the same change.

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