ROTATIONAL BANDS OF EVEN-EVEN AXIALLY SYMMETRIC NUCLEI

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It has been shown by Davydov and Filippov that by using the Hamiltonian obtained by Ford in averaging the interaction between the external nucleons and the nuclear core, one can write the equation for the collective motion of an axially symmetric even-even nucleus with total angular momentum \( \mathbf{J} \) in the form

\[ \frac{dU}{dJ} = -2U_J \frac{dJ}{dJ} + 2U_J = 0, \]

where \( U_J \) satisfies the boundary condition

\[ U_J(\infty) = 0, U_J(0) \rightarrow e^{-\eta J}, \]

for \( \eta \rightarrow \infty. \)

The eigenvalue \( \eta \) of Eq. (1) is not in general an integer, and determines the energy \( \epsilon_p(J) \) of the collective nuclear motion by the equation

\[ \epsilon_p(J) = \frac{1}{2} \left( \frac{J+1}{J+1} \right) \frac{U(J+1)}{U(J)} \left( \frac{J+1}{J+1} \right)^{\eta} \]

\[ + \frac{J(J+1)}{6\hbar^2} + \frac{J(J+1)}{2\hbar^2} \]

\[ - \frac{U(J)}{U(J)} \left( \frac{J+1}{J+1} \right)^{\eta}. \]

Thus the energy of nuclear collective motion for each value of \( J = 0, 2, 4, \ldots \) is determined uniquely by just the two parameters \( \epsilon_0 \) and \( \eta \), which are related to parameters of Bohr and Mottelson's model. Davydov and Filippov \( ^2 \) investigated the solutions of Eq. (1) for the case \( \delta < 1 \). In this way we present the results of a solution of this set of equations for the case \( \delta > 1 \).

The figure gives a graph of \( \epsilon_p(J)/\hbar \omega_0 \) vs. \( \delta \); the numbers on the curves give the values of \( \eta \). It is seen from the figure that when \( \delta > 2.5 \), the energy spectrum of collective excitations of even-even nuclei breaks up into a set of rotational-vibrational bands for certain nuclei with the experimental data. We also give the values of \( \eta_{0\delta} \) and \( \delta \) which have been used to calculate the theoretical excitation energy.

In Table 2 we give the \( \delta \) dependence of the energy ratios of the first and second \( (1 = 2) \) rotational-state sublevels in the first and second \( (1 = 2) \) rotational-state sublevels of the nuclear states. If the energy of collective oscillations is approximated in the form

\[ \epsilon = \eta \omega_0 + A(J + 1) - B(J + 1)^2, \]

\[ A = \delta^2/2\delta, \]

\[ B = (\delta_{0\delta})^{-1}(\delta/\delta^2), \]

Table 1

<table>
<thead>
<tr>
<th>Energy level (MeV)</th>
<th>( J = 0 )</th>
<th>( J = 2 )</th>
<th>( J = 4 )</th>
<th>( J = 6 )</th>
<th>( J = 8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theory</td>
<td>100.09</td>
<td>200.35</td>
<td>300.67</td>
<td>400.99</td>
<td>501.67</td>
</tr>
<tr>
<td>Experiment</td>
<td>100.09</td>
<td>200.35</td>
<td>300.67</td>
<td>400.99</td>
<td>501.67</td>
</tr>
</tbody>
</table>

Table 2

| \( J = 0 \) | \( J = 2 \) | \( J = 4 \) | \( J = 6 \) | \( J = 8 \) | \( J = 10 \) | \( J = 12 \) | \( J = 14 \) | \( J = 16 \) | \( J = 18 \) | \( J = 20 \) | \( J = 22 \) | \( J = 24 \) | \( J = 26 \) | \( J = 28 \) | \( J = 30 \) | \( J = 32 \) | \( J = 34 \) | \( J = 36 \) | \( J = 38 \) | \( J = 40 \) | \( J = 42 \) | \( J = 44 \) | \( J = 46 \) | \( J = 48 \) | \( J = 50 \) |
|-------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| Theory      | 1.45       | 1.70       | 1.95       | 2.15       | 2.30       | 2.45       | 2.60       | 2.75       | 2.90       | 3.05       | 3.20       | 3.35       | 3.50       | 3.65       | 3.80       | 3.95       | 4.10       | 4.25       | 4.40       | 4.55       | 4.70       | 4.85       | 5.00       | 5.15       | 5.30       | 5.45       | 5.60       |
| Experiment | 1.45       | 1.70       | 1.95       | 2.15       | 2.30       | 2.45       | 2.60       | 2.75       | 2.90       | 3.05       | 3.20       | 3.35       | 3.50       | 3.65       | 3.80       | 3.95       | 4.10       | 4.25       | 4.40       | 4.55       | 4.70       | 4.85       | 5.00       | 5.15       | 5.30       | 5.45       | 5.60       |

MESONIUM AND ANTIMESONIUM

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GELL-MANN and Pais \( ^1 \) were the first to point out the interesting consequences which follow from the fact that \( \kappa_1 \) and \( \kappa_2 \) are not identical particles. \( ^2 \) The possible \( \kappa_1^2 \rightarrow \kappa_2 \) transition, which is due to the weak interactions, leads to the necessity of considering neutral K-mesons as a superposition of particles \( \kappa_1 \) and \( \kappa_2 \) having a different combined parity. \( ^3 \) In the present note the question is treated whether there exist other "mixed" neutral particles (not necessarily "elementary") besides the \( \kappa_2 \)-meson, which differ from their anti-particles and for which the particle-antiparticle transitions are not strictly forbidden.

On the conservation of the number baryons and light fermions (or as sometimes called, conservation of nucleon and neutrino charge) strongly limit the number of possible mixed neutral particles. Because of the first-mentioned law mixed particles cannot occur amongst the baryons (e.g. a neutron; a hydrogen atom etc.) and because of the second law of such particles cannot exist among the light particles with only one fermion (e.g. neutrino, the systems \( \nu^+\nu^- \) and \( \bar{\nu}^+\bar{\nu}^- \), etc.).

From this it evidently follows that besides the \( \kappa_2 \)-meson the only system consisting of presently-known constituents which could be a mixed particle would be mesonium, defined as the bound system of \( \nu^+\nu^- \) Antimesonium, i.e., the system of \( \bar{\nu}^\nu \) clearly is different from mesonium and, furthermore, the
mesonium $\rightarrow$ antimeson inversion is not only not forbidden by any of the known laws, but actually should occur by virtue of already established interactions.

Indeed, the transitions

$$e^{-}\gamma e^{+} \rightarrow (\pi^{+} \pi^{-}) \rightarrow (\pi^{+}\pi^{-})$$

(1)

would be induced by the same mechanism that is responsible for the decay of the $\mu$-mesons. The probability $1/\bar{\sigma}$ of the real decay process

$$e^{-}\gamma e^{+} \rightarrow \pi^{+} \pi^{-} \gamma$$

(2)

which can be easily obtained by taking into account the size of the mesonium, is found to be $10^{-4}$ sec$^{-1}$, i.e., this probability is $10^{8}$ times smaller than the usual decay probability of the $\mu$- meson. It is therefore impossible in practice to observe this process, which would be indicated by a track corresponding to a stopping $\mu$- meson which decays without the emission of a decay electron.

The inversion time, process (1), is proportional to $\frac{1}{\bar{\sigma}}$ and is determined by the mass difference $\Delta m$ between the systems symmetrical and antisymmetrical with respect to mesonium and antimeson. This mass difference is proportional to the first power of the matrix element of the mesonium $\rightarrow$ antimeson transition. If this transition is due to a process involving two consecutive transitions, as in (1), $\Delta m$ is proportional to the square of the coupling constant. The inversion time is then of the order $\theta$ and $10^{8}$ times longer than the half life of the $\pi$-meson ($\tau = 2 \times 10^{-10}$ sec). The meson half life then determines also the mesonium half life.

However, if one admits a direct $(\mu^{+}\pi^{-})/(\mu^{-}\pi^{+})$ interaction, then the inversion time $T$ can be very significantly shorter than $\theta$. Indeed, then the mass difference $\Delta m'$ between the symmetrical and the antisymmetrical system $(\Delta m' \approx 2M/\sqrt{c})$ where $M$ is the transition matrix element $\pi$ is proportional to the first power of the coupling constant $g$. We consequently have:

$$T = \frac{\Delta m'}{\pi(2g/c)^{2}}$$

where $\tau$ is the radius of the mesonium. Assuming that the direct interaction $(\mu^{+}\pi^{-})/(\mu^{-}\pi^{+})$ has the same strength as the other weak interactions, we get $g \approx 3 \times 10^{-4}$ sec$^{-1}$ and $T \approx 2 \times 10^{-10}$ sec, which is only 300 times longer than $\tau$. Under these conditions it seems at first glance that the mesonium $\rightarrow$ antimeson inversion should be observable without too great difficulty. For example, one should see a "fast" negative electron after stopping a $\mu$- meson according to the process $(\mu^{+}\pi^{-})/(\mu^{-}\pi^{+}) - e^{+} \pi^{+} + \pi^{-} + e^{-}$.

Unfortunately, however, the inversion mesonium $\rightarrow$ antimeson cannot take place inside of matter, owing to the electrical charge asymmetry of the nucleons. This leads to a difference of the mass of mesonium and antimeson in matter. Furthermore, it should be pointed out that the probability of emission of a fast negative electron (in vacuo) is proportional to $(\pi/\tau)^{2}$ and not to $(\tau/\pi)^{2}$. Denoting by $e_{\mu}^{+}(t)$ and $e_{\mu}^{-}(t)$ the probability of finding a mesonium and antimeson respectively in vacuum at time $t$ if one mesonium "atom" exists at time zero, then

$$e_{\mu}^{+}(t) \sim \frac{1}{\sqrt{\tau}} \sqrt{1 + \frac{c^{2}}{\tau^{2}}} \quad \text{and} \quad e_{\mu}^{-}(t) \sim \frac{1}{\sqrt{\tau}} \sqrt{1 - \frac{c^{2}}{\tau^{2}}}$$

where the half life of mesonium and antimeson was assumed to be the same and equal to the $\mu$-meson half life. For these initial conditions the emission probability of a positive and negative electron respectively in vacuo is given by

$$P(e^{+}) = \frac{1}{\sqrt{\tau}} \sqrt{1 + \frac{c^{2}}{\tau^{2}}} \quad \text{and} \quad P(e^{-}) = \frac{1}{\sqrt{\tau}} \sqrt{1 - \frac{c^{2}}{\tau^{2}}}$$

If there would exist in nature charged particles of long half life with small nuclear interaction, then an effect analogous to the one presently described could be observed. The half life of the particles of mass $\approx 500 m_{e}$ observed by Alkibian et al. has not been determined yet; it is merely known to be much greater than $5 \times 10^{-9}$ sec.

It was assumed above that there exists a conservation law for the neutrino charge, according to which a neutrino cannot change into an antineutrino in any approximation. This law has not yet been established; evidently it has been merely shown that the neutrino and the antineutrino are not identical particles. If

* The analogous case of the $K^{+} \rightarrow K^{0}$ transition to first order in the weak interaction has been treated in detail in Ref. 7.