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Quantifying the reheating temperature of the universe

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The aim of this paper is to determine an exact definition of the reheat temperature for a generic perturbative decay of the inflaton. In order to estimate the reheat temperature, there are two important conditions one needs to satisfy: (a) the decay products of the inflaton must dominate the energy density of the universe, i.e. the universe becomes completely radiation dominated, and (b) the decay products of the inflaton have attained local thermodynamical equilibrium. For some choices of parameters, the latter is a more stringent condition, such that the decay products may thermalise much after the beginning of radiation-domination. Consequently, we have obtained that the reheat temperature can be much lower than the standard-lore estimation. This result has an immediate impact on many applications which rely on the thermal history of the universe.

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I. INTRODUCTION

The transition from a cold inflating universe to a hot thermal universe depends *solely* on the inflaton mass, m_ϕ, its coupling α_ϕ to the relevant degrees of freedom (d.o.f), and the dominant coupling between the decay products. In the case of Standard Model (SM) particles, it is predominantly the strong interaction, $\alpha_s \sim 1/30$. This epoch is known as reheating [1], or preheating [2] (for a review see [3]). In this paper we will mostly concentrate on the case where the inflaton has a small Yukawa coupling to the relevant d.o.f., which would typically yield a perturbative decay of the inflaton to its almost massless quarks, leptons and gluons. This is well justified for a SM *gauge singlet* inflaton, since the SM quarks and leptons are *chiral* in nature, and therefore the lowest order couplings are determined by the dimensional 5 operators in the potential, see [4]. Inflation could be driven by many independent sectors [5], but what matters is the last field which is responsible for finally reheating the universe in our patch for the success of Big Bang Nucleosynthesis (BBN) [6].

Especially, a SM gauge singlet inflaton could also couple to the SM Higgs with a 4-dimensional coupling, but through quartic coupling the inflaton never decays unless ϕ develops a VEV (vacuum expectation value): it rather leads to $\phi\phi \leftrightarrow HH$ scatterings, where ϕ is the inflaton and H denotes the SM Higgs. In order to deplete the inflaton quanta it is still important to rely on the perturbative decay of the inflaton [7] [1].

[1] Our treatment is very general and it can be applicable to supersymmetric theories. However there is a word of caution on how the inflaton couples to the supersymmetric Standard Model degrees of freedom, which depends very much on the origin of the inflaton. If inflaton is SM gauge singlet, see [8], if inflaton is SM gauge invariant field, such as one belongs to the supersymmetric flat directions of squarks and sleptons [9], see [10].

Typically, the reheating process is assumed to be instantaneous, with an efficient energy density conversion from the inflaton to the relativistic plasma. Within this framework the concept of reheating temperature T_{rh} has been defined, see [1, 11], ultimately relying on the assumption of the presence of local thermal equilibrium (LTE) at the very instant of conversion from the initial coherent oscillations of the inflaton domination to the radiation domination.

The aim of this work is to determine a proper definition of the reheat temperature of the universe keeping in mind when the LTE is established along with the fact that the inflaton has completely decayed into radiation. When and how should we evaluate the reheat temperature is an important question for a number of applications ranging from evaluating the baryonic asymmetry, dark matter abundance and the success of BBN [11]. In this paper we shall put down the criteria of estimating the reheat temperature, based on when the inflaton decay products attain their thermalisation. Depending on whether the decay products of the inflaton thermalise before or after the radiation has dominated the universe, the reheat temperature will be very different. In either situation the notion of reheat temperature only makes sense when the universe is completely dominated by the radiation bath.

If thermalisation of the ambient plasma occurs during the coherent oscillations of the inflaton, one may be able to associate a maximum temperature for the relativistic species [11, 12], but if the thermalisation time scale is longer than that of the inflaton-to-radiation domination transition time scale, the notion of temperature does not make sense until the universe reaches its full LTE.

This paper is organised as follows. In section II we set the stage and write down the relevant equations for our analysis. The standard lore about the reheating epoch is briefly commented in section III. Section IV is devoted to present our analysis, in which we study the conditions under which the plasma attains thermalisation. Later on, in section V we discuss the concept of reheat temperature such as to properly capture the issues of thermalisation. Finally, we conclude in section VI.

II. KEY ASSUMPTIONS AND EQUATIONS

For the sake of simplicity, we will assume *universal* inflaton coupling, α_ϕ, to all its decay products, determined by the number of relativistic d.o.f. g_*. Since the decay products of the inflaton are light, just from kinematics, they will typically have an initial momentum roughly given by: $m_\phi/2$ for two-body decay, or $m_\phi/3$ for a three-body decay processes. The inflaton is assumed here to be a SM gauge singlet - it will decay universally to all its decay products, i.e. all the relativistic species g_* would be excited.

Once the decay products are all excited there are two important processes which lead to thermalisation of all the d.o.f., or establish a LTE. Whereas a detailed thermalisation analysis of the plasma is out of the scope of this paper, some of its features are essential to our analysis, see Refs. [13, 14]:

1. *Kinetic equilibrium*: Redistribution of the momentum between different decay particles. This can be achieved by number conserving $2 \to 2$ scatterings with gauge boson exchange in the t-channel [13, 14].

2. *Chemical equilibrium*: Number violating $2 \to 3$ scatterings with gauge boson exchange in the t-channel are required to establish the chemical equilibrium [13, 14]. Higher order process are suppressed by further powers of the gauge coupling. Typically $2 \to 3$ interaction rate is higher than that of $2 \to 2$.

We will be using $2 \to 3$ for estimating the thermalisation rate, Γ_{th}, and various time scales. A typical example would be a quark-quark scattering with a gluon as a final-state radiation: $qq \to qqg$, whose typical cross-section is dominated by the momentum of the ambient relativistic plasma [14]:

$$\sigma \sim \frac{\alpha_s^3}{p(t)^2} \log\left(\frac{m_\phi^2}{p(t)^2}\right) , \tag{1}$$

where $\alpha_s \sim 1/30$ is the typical strong gauge coupling of the SM, and $p(t)$ is the 3-momentum transferred in the scattering.

The evolution of the inflaton, and the relativistic decay product's energy densities during the reheating period is described by the coupled set of Boltzmann equations, see [11]:

$$\begin{cases} \dot{\rho}_\phi + 3H(t)\rho_\phi = -\Gamma_\phi \rho_\phi \\ \dot{\rho}_R + 4H(t)\rho_R = \Gamma_\phi \rho_\phi + \Gamma_{\rm th}(\rho_R - \rho_R^{\rm eq}) , \end{cases} \tag{2}$$

where the dots denote derivatives w.r.t. the physical time, $\rho_\phi(\rho_R)$ is the energy density of inflaton (radiation), being $\rho_R^{\rm eq}$ the equilibrium one; $H(t)$ is the Hubble parameter accounting for the expansion of the universe; $\Gamma_\phi \equiv \alpha_\phi m_\phi$ is the inflaton decay rate, and $\Gamma_{\rm th}$ is the reaction rate responsible for thermalisation of the radiation plasma. Of course, once LTE is attained, the direct and inverse interactions among relativistic species counterbalance each other and the evolution of ρ_R is dictated solely by the inflaton source and the Hubble expansion.

III. ASSUMING LTE IS ESTABLISHED SOON AFTER INFLATON DECAY

Previous works which are relevant to our study have assumed LTE while studying the evolution of the relativistic species during the reheating period [11], see however [12–14, 17] for emphasising the importance of acquiring LTE. At any epoch during reheating, as long as there is a relativistic bath in thermal equilibrium, we can extract an instantaneous temperature as:

$$T(t) = \left[\frac{30}{\pi^2}\rho_R(t)/g_*(t)\right]^{1/4} \tag{3}$$

For a constant g_* during the whole period, the evolution of the temperature according to Eq. (3) is such that it has a maximum $T_{\rm max}$ [11, 12], which can be estimated as:

$$T_{\rm max} \simeq \left[\frac{1.57}{\pi^3 g_*}\right]^{1/4} \sqrt{M_P}\,(\Gamma_\phi H_I)^{1/4} , \tag{4}$$

being H_I the initial Hubble rate. Indeed, $T_{\rm max}$ can be potentially much larger than the reheating temperature, T_{rh}. The latter is usually defined as the temperature of the plasma assuming an instantaneous conversion of the inflaton's energy density into radiation, at the time when $H(t) \approx \Gamma_\phi$, such that:

$$T_{rh} = \left(\frac{90}{8\pi^3 g_*}\right)^{1/4} \sqrt{\Gamma_\phi M_P} . \tag{5}$$

IV. WHEN IS LTE ATTAINED?

However, LTE has to be attained and should not be taken for granted from the onset of the inflaton decay. In our analysis *we do not assume LTE* as a given condition for the relativistic species. Instead, we evaluate when and for which region of the inflaton parameters, m_ϕ and α_ϕ for a fixed $\alpha_s = 1/30$, this condition is achieved.

As justified later, there are two regions of the parameter space (α_ϕ, m_ϕ), for which Eq. (2) can be simplified such that the term $\Gamma_{\rm th}(\rho_R - \rho_R^{\rm eq})$ can be safely discarded:

1. Very small α_ϕ and very large m_ϕ, for which $\Gamma_{\rm th}$ is very small:

$$\Gamma_{\rm th} \ll \Gamma_\phi \cdot \left(\frac{\rho_\phi}{\rho_R}\right), \quad \Gamma_{\rm th} \ll H \tag{6}$$

2. Very large α_ϕ and very small m_ϕ, for which $\rho_R \approx \rho_R^{\rm eq}$:

$$\Gamma_{\rm th} \gg \Gamma_\phi \cdot \left(\frac{\rho_\phi}{\rho_R}\right), \quad \Gamma_{\rm th} \gg H \tag{7}$$

We will justify the notion of very small and very large below. For these two cases, Eq.(2) simplifies to (working with a comoving coordinate, $x \equiv a(t) \times m_\phi$, where $a(t)$ is the scale factor) [12]:

$$\begin{cases} \frac{d\Phi}{dx} = -\left(\sqrt{\frac{3}{8\pi}}\frac{M_P}{m_\phi}\alpha_\phi\right)\frac{x\Phi}{\sqrt{R+x\Phi}} \\ \frac{dR}{dx} = \left(\sqrt{\frac{3}{8\pi}}\frac{M_P}{m_\phi}\alpha_\phi\right)\frac{x^2\Phi}{\sqrt{R+x\Phi}} \end{cases} \tag{8}$$

with

$$\Phi \equiv \rho_\phi m_\phi^{-4} x^3, \; R \equiv \rho_R m_\phi^{-4} x^4 \; . \tag{9}$$

The initial condition is:

$$R(x_I) = 0, \quad \Phi_I \equiv \Phi(x_I) = \frac{H_I^2 M_P^2}{8\pi/3} * m_\phi^{-4} x_I^3 \; , \tag{10}$$

where the subindex I refers to initial values. In many inflationary scenarios it is a good approximation to take $H_I \sim m_\phi$.

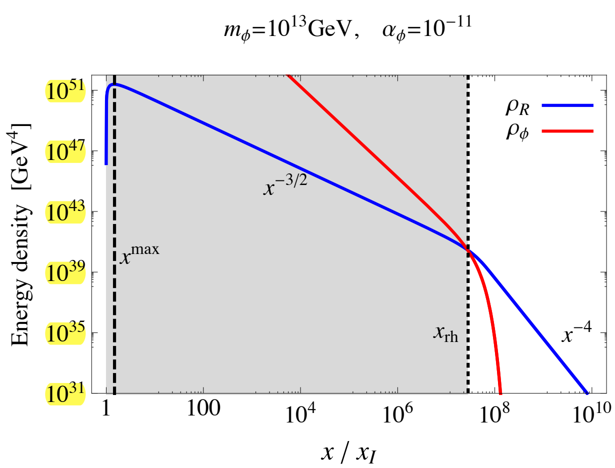

FIG. 1. Radiation energy density (blue line), ρ_R, and inflaton energy density (red line), ρ_ϕ, as a function of the scale factor, for $m_\phi = 10^{13}$ GeV and $\alpha_\phi = 10^{-11}$. The power laws indicate the behaviour of ρ_R in the different regimes. The region in grey represents the reheating epoch, which by the *standard lore* finishes when radiation dominates the expansion (see text for details).

We have solved Eq. (8) numerically, and the result is shown in Fig. 1, where for illustration we have taken $m_\phi = 10^{13}$ GeV and $\alpha_\phi = 10^{-11}$. We can infer that the radiation energy density (blue line) peaks very fast, around $x = x_{\rm max} \sim 1.5 x_I$, followed by a dilution due to the expansion. The position of the maximum is independent of the inflaton parameters. We also show for reference the inflaton energy density (red line), which as we can see completely dominates the expansion of the universe until the end of the reheating epoch. Analytically, during the inflaton-dominated period the radiation energy density goes like:

$$\rho_R^{id}(x) \approx \frac{2}{5}\sqrt{\frac{3}{8\pi}}\Gamma_\phi m_\phi^2 M_P \sqrt{\Phi_I}\; x^{-3/2}, \; x_I \ll x < x_{rh}$$
$$\approx \frac{0.15}{\pi} M_P^2 \; m_\phi^2 \alpha_\phi \left(\frac{x_I}{x}\right)^{3/2} \tag{11}$$

whereas for radiation-domination the expected x^{-4}-law is recovered:

$$\rho_R^{rd}(x) \approx \rho_R^{id}(x_{rh})\left(\frac{x_{rh}}{x}\right)^4, \qquad x_{rh} < x \; . \tag{12}$$

Here x_{rh} (to be computed below) encodes the moment at which reheating ends. The super-indices (id) and (rd) stem for (inflaton-domination) and (radiation-domination), respectively.

The condition under which the plasma enters in thermal equilibrium can be naively estimated by the requirement

$$\Gamma_{\rm th} = n_R(x)\langle\sigma(x)v\rangle > H(x) \; , \tag{13}$$

where we approximate the cross-section σ by Eq. (1), $v \approx c$ for relativistic species, and $n_R(x)$ is the relativistic number density. The latter can be directly extracted by solving Eq. (8) in terms of number densities instead of energy densities. Assuming 2-body decays of the inflaton (our results will not be affected much if we assume 3-body decay of the inflaton), see also [14]:

$$n_R(x) \approx 2n_\phi^I\left[1 - e^{\left(-\Gamma_\phi \int_{x_0}^x \frac{d\tilde{x}}{\tilde{x}\cdot H(\tilde{x})}\right)}\right]\left(\frac{x_I}{x}\right)^3 \tag{14}$$

where the initial inflaton number density, $n_\phi^I \sim \rho_\phi^I/m_\phi$, as well as $H(x)$, are computed according to the our numerical solution of Eq. (8). Analytical estimations of Eq. (14) can be obtained, as for the case of ρ_R, in two regimes. During inflaton-domination, $R(x)$ gives a negligible contribution to the Hubble rate, whereas Φ remains approximately constant, $\Phi \approx \Phi_I$. In this case, it is straightforward to obtain:

$$n_R^{id}(x) \simeq 2n_\phi^I(1 - e^{-\kappa x^{3/2}})\left(\frac{x_I}{x}\right)^3 \approx 2n_\phi^I \; \kappa \; x_I^3 \; x^{-3/2}$$
$$\simeq \frac{0.5}{\pi} M_P^2 \; m_\phi \alpha_\phi \left(\frac{x_I}{x}\right)^{3/2} \; , \tag{15}$$

with $\kappa = (2/3)\alpha_\phi/x_I^{3/2}$. On the other hand, for radiation-domination we clearly have:

$$n_R^{rd}(x) \simeq 2n_\phi^I\left(\frac{x_I}{x}\right)^3 = \frac{3}{4\pi}M_P^2 \; m_\phi \left(\frac{x_I}{x}\right)^3 \; . \tag{16}$$

The value x_{rh} at which the regime changes could be computed in several ways, one of which is demanding $n_R^{id}(x_{rh}) = n_R^{rd}(x_{rh})$, resulting in:

$$x_{rh} = \kappa^{-2/3} \simeq 1.3 \; x_I/\alpha_\phi^{2/3} . \tag{17}$$

Note that this value is independent of m_ϕ - heavier inflaton would have a shorter lifetime, but at the same time they would cause a faster expansion rates at early times.

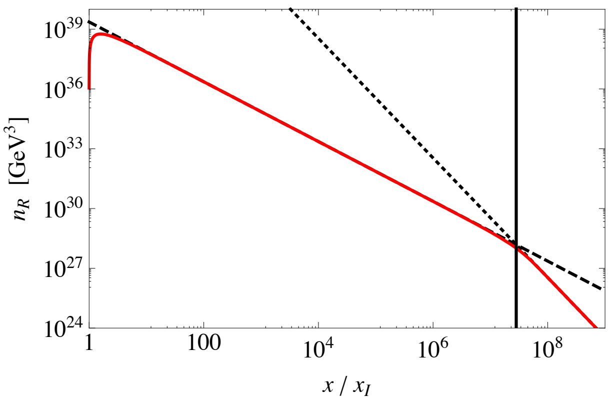

FIG. 2. Radiation number density as a function of the scale factor, for $m_\phi = 10^{13}$ GeV and $\alpha_\phi = 10^{-11}$. The solid red line is the solution of Eq. (14), where $H(x)$ is computed numerically from Eq. (8). Dashed black line is the solution in Eq. (15), whereas the dotted black line is the solution in Eq. (16). The solid black vertical lines is the value of x_{rh} according to Eq. (17).

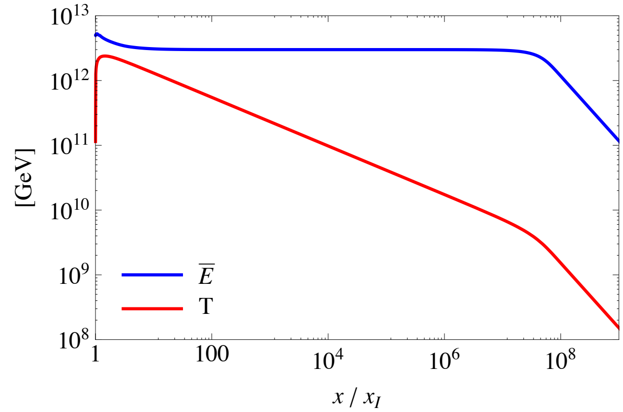

FIG. 3. Comparison of the mean kinetic energy as computed according to Eq. (18), and the temperature assuming LTE from the very onset of the inflaton decay as in Eq. (3), for $m_\phi = 10^{13}$ GeV, $\alpha_\phi = 10^{-11}$. One can see the obvious distinction and the importance of understanding when one should associate a temperature to the decay products of the inflaton.

We have shown in Fig. 2 the perfect agreement of the analytical estimations made in Eqs. (15-17) w.r.t. the numerical solution in Eq. (14).

Coming back to the thermalisation analysis, since we cannot rely on an equilibrium distribution at this point, we take the momentum $p(x)$ in Eq. (1) to be:

$$p(x) = \frac{d\rho_R(x)}{dn_R(x)} = \frac{d\rho_R(x)}{dx} \cdot \left[\frac{dn_R(x)}{dx}\right]^{-1} . \qquad (18)$$

This expression directly follows from the definitions of n_R and ρ_R, *without* assuming any particular shape of the distribution function $f(p)$. We would like to emphasise here that Eq.(3), in the absence of LTE, should not even have an interpretation of mean kinetic energy, since its functional shape incorporates the assumption of LTE-like $f(p)$.

Taking then Eq. (18) as a proper measure of the mean kinetic energy $\bar{E}$ of particles in the plasma, we compare $\bar{E}(x)$ with the temperature $T(x)$, extracted from Eq. (3) under the assumption of thermal equilibrium. This is shown in Fig. 3. As can be observed, $\bar{E}$ is constant over almost the whole reheating period, $\bar{E}^{id} \approx m_\phi/3$, whereas after reheating its evolution follows the same law as for $T(x)$, i.e. the well-known $T \propto x^{-1}$ behaviour of the radiation-dominated universe, resulting in:

$$\bar{E}(x)^{rd} \simeq \frac{0.5\ m_\phi}{\alpha_\phi^{2/3}} \frac{x_I}{x} . \qquad (19)$$

Physically it makes sense: during inflaton-domination the plasma (containing the relativistic species from the inflaton decay) is getting constantly reheated by the inflaton decay, and it turns out that it does so at a rate which is equal to the cooling rate due to the expansion. Afterwards, when the inflaton has decayed completely and only radiation remains, the energy of the relativistic species gets only redshifted by the expansion of the universe.

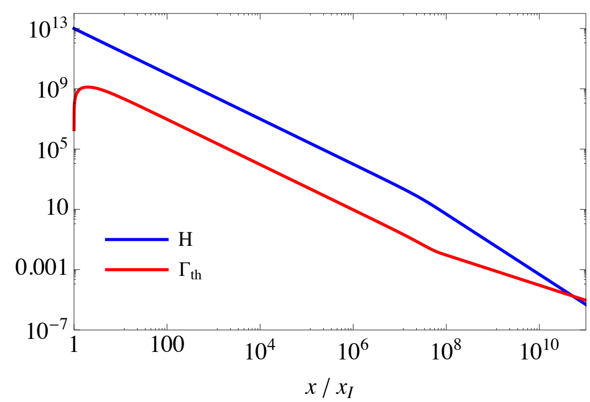

FIG. 4. Reaction rate Γ as a function of x, in magenta, as compared to the Hubble parameter (red). Break in the slopes determine the transition from inflaton domination to the radiation domination. Note that thermalisation time scale is larger than the matter-to-radiation transition scale.

As for the thermalisation condition is concerned, depending on the value of (α_ϕ, m_ϕ) it can be attained during inflaton-domination or afterwards, during radiation-domination. We should evaluate $\Gamma_{\rm th}$ by making use of the mean energies, $\bar{E}$, instead of the *temperature*, since as we pointed out above - we cannot rely at this point on thermal distribution. We then compare $\Gamma_{\rm th}$, according to the case, with:

$$H(x) \approx \begin{cases} 2.9\ m_\phi \left| \dfrac{0.6 x_I^{3/2}}{\sqrt{\pi} x^{3/2}} - 0.1\alpha_\phi \right|, & id \text{ epoch} \\ \\ 1.6 \dfrac{m_\phi}{\sqrt{\pi}\alpha_\phi^{1/3}} \dfrac{x_I^2}{x^2}, & rd \text{ epoch} \end{cases} \tag{20}$$

where in inflaton-domination denoted here by id, the Hubble rate is approximately $H \propto (\rho_\phi^{id})^{1/2}$, whereas in radiation-domination denoted here by rd, $H \propto (\rho_R^{rd})^{1/2}$.

We have shown in Fig. 4 the comparison of $\Gamma_{\rm th}$ and H from the numerical solution of Eq. (8), using $\alpha_\phi = 10^{-11}$ and $m_\phi = 10^{13}$ GeV, for the sake of illustration. As we note, the evolution of $\Gamma_{\rm th}$ is parallel to that of H for nearly the whole reheating period. Indeed, in this region $\sigma(x)$ is nearly constant (because $\bar{E}$ is) and thus $\Gamma_{\rm th}$ scales as $n_R(x)$, the latter evolving as $\rho_R(x)$ as was already deduced above (see Eqs. (11) and (15)). On the other hand the Hubble rate, even if dominated by the inflaton oscillations, also evolves as $\rho_R(x)$ [2]. It is only after the inflaton population decreases substantially that the universe starts being radiation-dominated, thus the thermalisation processes become faster than the expansion rate and thermal equilibrium is achieved. The numerical solution for the thermalisation time, x_{th}, is around $x_{th} = 5 \times 10^{10} x_I$ for this choice of parameters.

Analytically it is possible to obtain the value of x at which the thermalisation occurs, $\Gamma_{\rm th}(x_{th}) = H(x_{th})$. We just need to build up $\Gamma_{\rm th}$ from Eqs. (16),(19) and (1), whereas the Hubble rate is approximated by Eq. (20).

For the sake of simplicity, assuming a total thermalisation cross-section which goes like $\sigma_{th} = \alpha_s^3/E^2$, see Eq. (1), we obtain the following solution for x_{th}:

$$\frac{x_{th}^{rd}}{x_I} \approx \frac{m_\phi^2}{\alpha_s^3\ M_P^2 \alpha_\phi^{5/3}} \tag{21}$$

for the case of a radiation-dominated thermalisation.

On the other corner of the parameter space, for large α_ϕ and small m_ϕ, it usually happens that thermalisation happens very fast, $x_{\rm th}^{id} \lesssim x_{\rm max}$, when the inflaton still dominates the expansion.

This is one of the main results of our analysis: for some choices of the pair (α_ϕ, m_ϕ), the plasma does not reach thermalisation at the time when the universe becomes radiation-dominated, but later. This happens for:

$$\alpha_\phi \lesssim \left(\frac{1}{\alpha_s^3}\right)\left(\frac{m_\phi}{M_p}\right)^2 . \tag{22}$$

As an example for illustration, for a heavy mass, $m_\phi = 10^{14}$ GeV, thermalisation reactions driven by $2 \to 3$ processes of strong gauge coupling (as in Eq. (1)), the relativistic species reaches thermal equilibrium later than the beginning of the radiation-domination era as long as $\alpha_\phi \lesssim 10^{-6}$.

Physically speaking this can be understood as follows: even when the universe starts to become dominated by the radiation energy density, the thermalisation reaction rates may still be inefficient because of the very large typical energies of the interacting particles, $\bar{E} \lesssim \mathcal{O}(m_\phi)$, inherited from the inflaton decays and almost unaffected otherwise (see Fig. 3, for an illustrative point). These large energies penalise the cross-sections, until the redshift is important enough as for the scattering process to become efficient enough, such that $\Gamma_{\rm th} > H$ and LTE is finally attained.

[2] This can be deduced from Eq. (10) under the assumption of $\Phi \approx$const.

V. DEFINITION OF REHEAT TEMPERATURE

Now let us define the reheating temperature, T_{rh}, as computed according to energy density (cf. Eq.3), provided the radiation have just thermalised, and dominates the Hubble expansion rate of the universe.

$$T_{rh} = T(x), \quad x = \max(x_{th}, x_{rh}) . \tag{23}$$

There are two cases of interest:

1. **Instant thermalisation -** ($x_{th} \ll x_{rh}$): Thermalisation of relativistic species is attained almost instantaneously (usually even around x_{max}), already during the coherent oscillations of the inflaton, and they maintained LTE throughout reheating and also at the time when the universe becomes radiation dominated. Following our prescription in (23), in this case the reheat temperature is determined by:

$$T_{rh}(x_{th} \ll x_{rd}) \approx \frac{0.6}{g_*^{1/4}} \sqrt{\alpha_\phi\ m_\phi\ M_P}\ ; \tag{24}$$

   A couple of points to note: we see that $T_{rh}(x_{th} \ll x_{rd})$ behaves exactly as the usual T_{rh} of instant-reheating scenario, see Eq. (5), with an $\mathcal{O}(1)$-difference in a prefactor. Indeed $T_{rh}(x_{th} < x_{rd})$ is a bit smaller than the usual definition of reheating case, as assumed in Eq. (5), since the latter corresponds to a maximal thermalisation-efficiency by definition. In our case, the lower efficiency translates into a bit smaller reheating temperature (see Fig. 5 below). On the other hand, in this scenario, it is indeed possible to define a maximum temperature of the relativistic species, $T_{max} \equiv T(x_{max}) > T_{rh}$.

2. **Delayed thermalisation -** ($x_{th} \gg x_{rh}$): Thermalisation happens deep inside the radiation dominated era, such that the reheat temperature is determined by:

$$T_{rh}(x_{th} \gg x_{rd}) \approx \frac{0.7\ \alpha_s^3\ \alpha_\phi^{3/2} M_P^{5/2}}{g_*^{1/4} m_\phi^{3/2}} . \tag{25}$$

Note that $T_{rh}(x_{th} \gg x_{rd})$ has an opposite behaviour with respect to m_ϕ. This is the most important result of our work - in some region of the parameter space (α_ϕ, m_ϕ), where thermalisation happens after radiation starts dominating, the reheating temperature actually decreases with the inflaton mass. Physically this is due to the following. For larger m_ϕ, larger is the mean energy $\bar{E}$ of the relativistic species. This penalises the cross-sections for thermalisation reactions, rendering the thermalisation rate less efficient at the end of the day, which is attained later. Consequently this lowers down $T_{rh}(x_{th} \gg x_{rd})$.

A closer inspection of Fig. 5 reveals some values of m_ϕ and α_ϕ for which the numerical results are not shown. These "holes" in the scan are due to the limited validity of our numerical solution of Eq.(8). As discussed above this expression, the $\Gamma_{\rm th}(\rho_R - \rho_R^{\rm eq})$ term is important when $\Gamma_{\rm th}$ becomes essentially comparable in size to the Hubble expansion and the inflaton source. Otherwise, either radiation-to-radiation terms are very inefficient (such that they do not play a role in the ρ_R-evolution), or if they are too efficient (such that production and annihilation balance each other in an equilibrium distribution), the Eq. (8) is a reasonable simplification of the original Boltzmann set of equations Eq. (2).

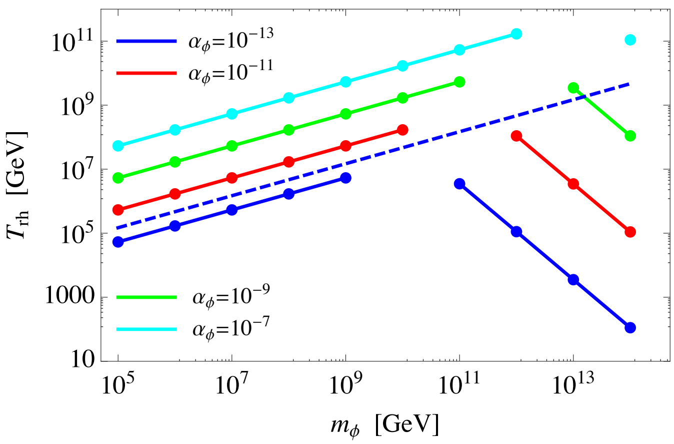

FIG. 5. Reheating temperature computed numerically by solving Eq.(8). This is represented with dots joined by solid lines for different values of α_ϕ: 10^{-13} (blue), 10^{-11} (red), 10^{-9} (green) and 10^{-7} (cyan), for $\alpha_s = 1/30$. The blue dashed-line is an incorrect depiction of reheating temperature (as in Eq. (5)), corresponding to $\alpha_\phi = 10^{-13}$.

We have shown in Fig. 5 the reheating temperature T_{rh} as a function of m_ϕ for different values of α_ϕ, computed numerically by solving Eq. (8) and represented with coloured dots joined by full lines. As commented above there are two regimes: one for which the thermalisation happens at inflaton-domination, where $T_{rh}(x_{th} < x_{rd})$ grows with m_ϕ and follows closely to $T_{rh} = (90/8\pi^3 g_*)^{1/4}\sqrt{\Gamma_\phi M_P}$ (see dashed blue line in Fig. 5); and a second regime for which the thermalisation happens deep inside radiation-domination era, where $T_{rh}(x_{th} > x_{rd})$ decreases with m_ϕ. Essentially, for the largest α_ϕ and the smallest m_ϕ, we are in the former regime, whereas for the smallest α_ϕ and the largest m_ϕ, we are in the latter regime.

Note that, for example, for $\alpha_\phi = 10^{-13}$ and $m_\phi = 10^{13}$ GeV the usual T_{rh} (as in Eq.(5)) largely overestimates the (more realistic) reheating temperature we have obtained in our analysis. For the second regime, where Eq. (25) applies, we can estimate the value of T_{rh} for these values, giving around 4×10^3 GeV, which is in very good agreement with the numerical result shown in Fig. 5.

VI. CONCLUSIONS AND DISCUSSIONS

In this work we have studied inflationary reheating, in particular revisiting the study of the thermalisation of the inflaton decay products from both an analytical and numerical point of view, by analysing the dominant thermalisation process of the relativistic plasma as a whole. We have solved the coupled set of Boltzmann equations in two clearly defined regimes: a) The $2 \to 3$ processes leading to thermalisation are too inefficient to affect the global evolution of the radiation energy-density itself, as a result the universe could be radiation dominated, but still not in local thermodynamical equilibrium (LTE), and b) when thermalisation process is very quick, at much larger rates compared to the Hubble expansion and the inflaton decay rate, in such a way that LTE of the decay products is attained very fast. In both regimes the Boltzmann equations are simplified in a similar fashion.

We have obtained the following important results:

- For sufficiently small α_ϕ and sufficiently large inflaton-mass m_ϕ, the relativistic plasma does not thermalises at the time where radiation-domination era begins, but (in some cases, much) later. When α_ϕ is very small there are not enough relativistic species at the matter-to-radiation transition to immediately thermalise, whereas for very large m_ϕ, the species are too energetic as for the relevant scattering processes to be efficient enough.

- We have determined a proper definition of the reheat temperature, in a generic scenario of perturbative decays of the inflaton. Essentially, two necessary conditions have to be eventually fulfilled: the plasma have to attain LTE, and it must dominate the expansion rate of the universe. This is such that for some region of the inflaton parameters (precisely the one commented in the first point), the reheat temperature turns out to be much smaller than the standard estimations.

Our results will have phenomenological implications for thermal leptogenesis (for a review see [15]), Affleck-

Dine baryogenesis, for a review see [9], gravitino abundance, for a review see [16], and dark matter creation during reheating [17]. More recently in the context of freeze-in mechanism through heavy portals [18–20], we have other examples of DM which are sensitive to the reheat temperature. Some of these topical issues might need to be reconsidered in light of the effects we have discussed in this paper.

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