Primordial abundance in the universe
&
Dark Matter detection

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Résumé

Après avoir détaillé comment sont survenus les principaux découplages dans l'histoire de l'univers primordial et comment nous pouvons utiliser l'équation de Boltzmann pour calculer les abondances actuelles, nous nous concentrerons sur le cas de la matière noire. Nous montrerons comment le rayonnement synchrotron peut être utilisé, avec l'équation de Boltzmann, pour contraindre la masse de particules candidates au titre de matière noire.

Abstract

After explaining how the main decoupling happened in the early universe and how the Boltzmann equation can be used to derive today's abundances we will turn our focus on the case of dark matter. We will show how the synchrotron radiation can be used, along with the Boltzmann equation, to constrain the mass of dark matter particles.

Table of contents

Introduction

One of the main occupation of these past decades has been improving our knowledge on the composition of the universe. Using the Cosmic Microwave Background (CMB), Supernovae type Ia (SNeIa), gravitational lensing, large scale structure and many other observables, scientific community came to the conclusion that the universe was made of 4% of baryonic matter, 0.2% of radiation (mostly photons and neutrinos), 23% of dark matter and 73% of dark energy. Learning more on dark matter and dark energy is considered a priority for the next decades. As an example, a partnership between NASA and the U.S. Department of Energy made a priority to investigate on dark energy through the JDEM project. As for dark matter, there are several projects investigating on it. We will come back on them in details in the third part and in Appendix C.

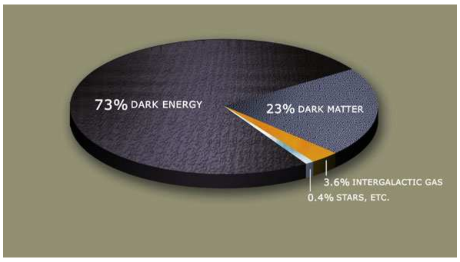

FIG. 1 – Contribution of the different component to the total density of the universe.

By hypothesis, dark matter doesn't interact otherwise than gravitically. As it doesn't interact through electromagnetic interaction, we cannot hope to recieve direct light from it. Since observation has been our main way to investigate our extraterrestrial environnement, how did we get hints that a quarter of our universe was made of such a matter ? We observed gravitational evidence or physical phenomena that made sense once supposing the existence of dark matter.

Dark Matter evidence

Galaxy rotation curve.

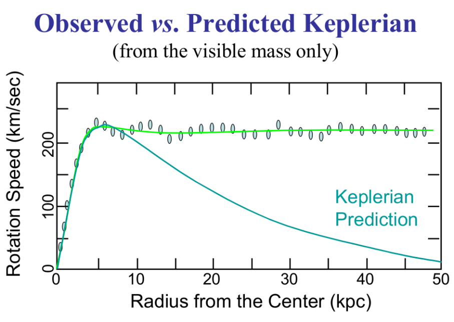

FIG. 2 – Typcal example of observed vs predicted galaxy rotation curve.

"The Galaxy Rotation problem" as it was known was first brought out by Fritz Zwicky in 1933. Zwicky worked at the California Institute of Technology as a professor of astronomy. Based on his observations of the Coma cluster, he compared the rotation speed of the galaxies to their mass estimated through their luminosity. His data indicated that galaxies are much more massive that what their light shows. He conjectured the existence of a matter that does not emit light : the dark matter. Zwicky's observation were confirmed and precised by Vera Rubin, a young and talented astronomer from the Carnegie Institution. An interesting anecdote is that Vera Rubin wasn't authorized to apply to Princeton University because “Princeton does not accept women”. That policy was not abandoned until 1975.

Cosmic Microwave Background.

Evidence of dark matter can be found in CMB[1] fluctuations[2]. We observe[3] $\frac{\Delta T}{T} \sim 10^{-5}$. Those fluctuations are proportional to the overdensties of baryonic gas δ_b. At photons decoupling, $\delta_b \sim 10^{-5}$. From that moment, galaxies and structures formation starts. At the begining of those formations, the density contrast is of the order of 180 when considering non linear anisotropies and the equivalent of the linear extrapolation is of the order of 1.5. Between the primordial fluctuations and the density contrasts observed at structure formation, there is a difference of, roughly speaking, 5 order of magnitude. In linear regime, density fluctuations increase proportionally to the scale factor, $a(t)$. Starting from a redshift $z_{CMB} = 1100$[4], this impose that the formation of the structure happened at a negative redshift, i.e. in the future. This being ridiculous, a possible explanation is to propose the existence of a massive fluid that doesn't interact with photons. The density fluctuations of that fluid may have increased before the photons decoupling : we suppose that these fluctuations reached 10^{-3} or 10^{-2} by the time of z_{CMB}. For more precision on the cosmology related to CMB, please refer to (*Challinor, 2009*).

Bullet cluster.

In the Bullet cluster, we can observe two clusters of galaxies that have quite recently passed right through each other. About 90% of ordinary matter in a cluster is intergalactic gas emitting hot X-ray and not the galaxies themselves. As the two clusters passed through each other, the hot gas in each smacked into the gas in the other, while the individulas galaxies and the dark matter passed right through. Indeed, dark matter is presumed to be collisionless. In appendix A, you can see how the hot gas is stuck in the middle of the two clusters that are now made of galaxies and dark matter which are continuing their way after passing through each other. As explained in detail in (*Clowe, 2006*) and in (*Bradac, 2006*), the fact that the gas stopped at the point of collision and stayed localised in the zone between the two clusters is an indication that there is a additional mass helping the galaxies to contain gravitationally the interacting gas. That additional mass supposidely being dark matter.

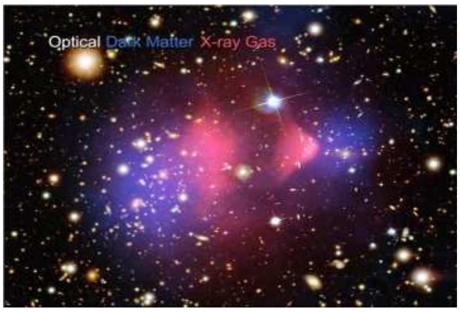

FIG. 3 – The Bullet Cluster : Dark Matter in blue and hot gas in pink.

Gravitational lensing.

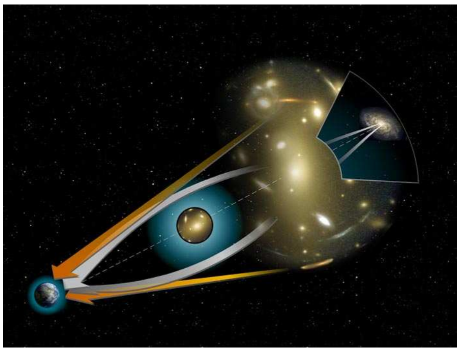

FIG. 4 – Gravitational lensing.

In astrophysics, a gravitational lens refers to a distribution of matter located between an observer and a distant source. This distribution of matter apply a strong gravtitational field around itself. As a result, the light travelling towards the observer will be bent. When observing this phenomenon for galaxy clusters as the matter distribution, we notice that the bending is stronger than expected for a cluster as massive as calculated through its luminosty. We then come to the conclusion that there is a strong component of matter that doesn't emit light in galaxy clusters : this would be dark matter.

[1]Even though Penzias and Wilson were awarded the Nobel Prize for the discovery of the cosmic microwave background radiation, it is Ralph Alpher that was the first along with his collaborator Robert Herman that were to predict its existence in 1948.

[2]See Appendix A for the image of the first 2 weeks light survey of PLANCK.

[3]rms value.

[4]See Appendix B for a brief history of the universe.

1. Visible Matter abundance

1.1. Usefull relation in cosmology

Time-Temperature correspondance.

In a thermal equilibrium regime where $T \ll m$ [derived from (*K*olb & Turner, 1990)] :

$$t \sim \left(\frac{T}{MeV}\right)^{-n} \equiv \left(\frac{E}{MeV}\right)^{-n} \equiv \left(\frac{E_0(z+1)}{MeV}\right)^{-n} \tag{1}$$

where $n = 2$ in radiation dominated era and $n = \frac{3}{2}$ in matter dominated era.

Hubble parameter.

The Friedmann equation gives :

$$H = \frac{\dot{a}}{a} = \sqrt{\left(\frac{8\pi G\rho}{3}\right)} \tag{2}$$

Where H is the Hubble constant. It is a function of the scale factor. It relates " the stretching " of the universe. Given that we consider the situation where $T \ll m$, the energy density is[5] :

$$\rho(T) = g_E(T)\frac{\pi^2}{30}T^4 \text{ where } g_E(T) = 2 + 6 \cdot \frac{7}{8}(\frac{T_\nu}{T_\gamma})^4 = 3.36 \tag{3}$$

Then, the expression of the Hubble parameter as a function of temperature is :

$$H(T) = \sqrt{(0.299 \cdot \pi^3 G T^4)} \tag{4}$$

From (*Kolb & Turner, 1990*), we know that the Hubble parameter as a function of mass is :

$$H(m) = 1.67 g_*^{\frac{1}{2}} \frac{m^2}{m_{pl}} \tag{5}$$

where : $g_* = \sum_{i=bosons} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_{i=fermions} g_i \left(\frac{T_i}{T}\right)^4$ is the degree of freedom.

1.2. Entropy

The entropy is a useful quantity that can be used to monitor the changes in an expanding universe. The expanding universe can be seen as a closed system for which the second law of thermodynamics can be writen as :

$$dU = TdS - pdV \tag{6}$$

Since ρ, the energy density[6] is a quantity often used in cosmology, equation (6) can be expressed as :

$$TdS = d((\rho + p)V) + Vdp \tag{7}$$

Using the comoving coordinates, the comoving volume is $V \equiv a^3$.

By substituting $dp = \frac{\rho+p}{T}dT$ in equation (7) and then by integrating, we are now able to express the entropy as a function of temperature : $S = a^3(\rho + p)/T$. From that, we can define the entropy density : $s \equiv \frac{S}{V} = \frac{\rho+p}{T}$.

1.3. Abundance of light species

All along this work, we will focus on a quantity, n, called abundance or number density. The abundance of a species is related to its density through $\rho = mn$. The definition of the number density on terms of phase space density gives :

$$n \equiv g e^{\frac{\mu}{T}} \int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} \tag{8}$$

[5] In part 1.3, we will come back on the ratio $\frac{T_\nu}{T_\gamma}$.

[6] $U \equiv \rho V$

where μ is the chemical potential.

At equilibrium :

$$n \equiv \begin{cases} g\left(\frac{mT}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m}{T}} \ , m \gg T \\ g\frac{T^3}{\pi^2} \ , m \ll T \end{cases} \tag{9}$$

Using equation (9), the number density of a species with charge Z and atomic number A is given by :

$$n_A = g_A \left(\frac{m_A T}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{(m_A-\mu_a)}{T}} \tag{10}$$

If we want to express that number density as a function of the neutron's and proton's abundance, we use the fact that $m_p \simeq m_n \simeq m_N$, $\mu_A = Z\mu_p + (A-Z)\mu_n$ and that the binding energy is $B_A = Zm_p + (A-Z)m_n - m_A$. The equation (10) becomes :

$$n_A = \frac{g_A A^{\frac{3}{2}}}{2^A} e^{\frac{B_A}{T}} n_n^{(A-Z)} n_p^Z \left(\frac{2\pi}{m_N T}\right)^{\frac{3(A-1)}{2}} e^{\frac{B_A}{T}} \tag{11}$$

Now that we know how to express the number density, we can use it to find out at which temperature particles freed from the plasma. Indeed , at fist, all particles interacted with each other at a very high energy and hence formed a plasma. As temperature decreased, either because of the ratio of mass over temperature ($e^{-\frac{m}{T}}$) or the ratio of binding energy over temperature ($e^{-\frac{B}{T}}$) in the number density or either because the cross section was decreasing, the interaction rate[7] failed to compensate the effect of the expansion of the universe. As a consequence, the mean free path of particles became at least the size of the universe[8] : meaning, the particles decouple from the plasma.

Neutron freeze out.

The advantage of following nuclear reaction at so early time is that all possible nuclei have binding energies less than typical photon energies. That way, only the weak interaction that causes conversion between protons and neutrons matters : $(p)+(e) \leftrightharpoons (n)+(\nu)$ and $(p)+(\vec{\nu}) \leftrightharpoons (n)+(e^+)$. Since protons and neutrons are a key element to the early reactions that took place in the early universe, the abundance of light elements that resulted from these early reactions is directly related to the neutron-proton ratio. In equilibrium, this ratio should vary as :

$$\frac{N_n}{N_p} = e^{-\frac{m_n - m_p}{T}} \tag{12}$$

From (*Peacock,1999*), we are assuming that the weak interaction switch off, freezing the neutron abundance at a temperature of : $T \simeq 1.39 \cdot 10^{10}$ K. Therefore, the equilibrium neutron-proton ratio is :

$$\frac{N_n}{N_p} = e^{-\frac{m_n - m_p}{T}} \simeq 0.34 \tag{13}$$

This result isn't completely accurate because the freeze-out condition was calculated assuming a temperature well above the electron mass threshold, whereas it appears that freeze-out actually occurs at about this critical temperature. In practice, the neutron abundance is lower. However, the equation (13) gives us a reference for the proton-neutron abundance when the early reactions occured.

Neutrino decoupling.

Neutrinos are kept in thermal equilibrium by the weak reaction : $(\nu)+(\bar{\nu}) \leftrightharpoons (e_-)+(e_+)$. The cross section of this interaction is $\sigma \sim G_F T^2$. G_F is the Fermi coupling constant. We are making the masseless particles approximation : $n \equiv T^3$. Therefore the interaction rate is : $\Gamma_\nu = n_\nu \sigma_\nu |v| = G_F T^5$. The equality between the interction rate we just gave and the Hubble parameter expressed in equation (4) is solved numerically and the result is :

$$T(\nu_{decoupling}) = 3.16 \cdot 10^{10}\ K = 2.73 \text{ MeV} \tag{14}$$

[7] In natural units, the interaction rate is the inverse of the mean free path.

[8] The size of the universe $\simeq$ the Hubble distance H^{-1}

Electrons and positrons decoupling.

When $T \ll m_e$, the eletrons and positrons fall out of chemical equilibrium with the photons because the density number of electrons and positrons is decreasing exponentially with the Boltzmann factor $e^{-\frac{m}{T}}$. Assuming $n_{e^-} = n_{e^+} = n_e$, the annihilation rate at thermal equilibrium is : $\Gamma_e = n_e(T)\frac{\pi\alpha}{m_e^2}$. When filling in the expression of the number density in that regime of equilibrium :

$$\Gamma_e \sim \alpha^2 \sqrt{\left(\frac{T^3}{8\pi m_e}\right)} e^{-\frac{m_e}{T}} \tag{15}$$

The end of chemical equilibrium happens when $H(T) = \Gamma_e(T)$. The numerical solving gives :

$$T(e_{decoupling}) \sim \frac{m_e}{40} = 148 \cdot 10^6\ K = 12.8 \text{ keV} \tag{16}$$

The decoupling is a process that conserves the entropy : $s = g_* T_\gamma = constant$. When the electrons and positrons are decoupling[a], the degree of freedom decreases and therefore the temperature rises. Physically, we say that electrons and positrons transfer some of their energy to the photons to maintain the entropy. At the time neutrinos decoupled, neutrinos and photons had the same temperature : $T_\nu = T_\gamma$. Since that decoupling, T_ν and T_γ followed the same law of evolution. But when the electrons and positrons gave their energy to the photons, the degree of liberty[b] changed for $g_* = \frac{11}{2}$ making T_γ getting bigger than T_ν.

[a]Nota Bene : we are still under the thermal equilibrium conditions

[b]See the degree of freedom as a function of temperature in Appendix B.

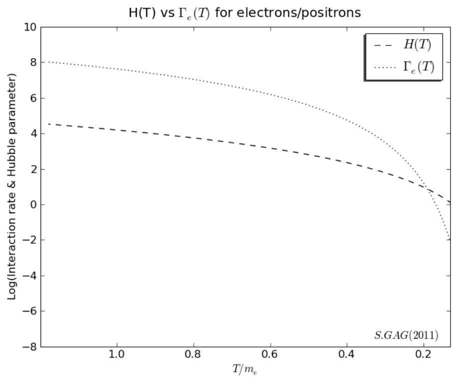

FIG. 5 – Expansion of the universe vs Interaction rate for electrons and positrons.

Finally, when the photons decouple, the degree of freedom change for $g_* = 2$ and thanks to the conservation of the entropy, we can write the following equalities :

$$T_\gamma^{\text{after } e^- \text{ decoupling}} = \left(\frac{11}{4}\right)^{\frac{1}{3}} T_\gamma^{\text{before } e^- \text{ decoupling}} \tag{17}$$

$$T_\nu^{\text{now}} = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma^{\text{now}} \tag{18}$$

That way, we can compute the photons and neutrinos temperature at any time from the temperature of the photons today. We used that relation in part 1.1 to compute an expression of the Hubble parameter as a function of temperature.

The first atoms.

We now want to find out at which temperature were formed the first atoms of hydrogen. To do so, we define our framework with the following assumptions :

- the universe is in thermodynamic equilibrium
- the universe is electrically neutral ($n_e = n_p$)
- what we call recombination is a proton and an electron combining to form a hydrogen atom in the ground state ($p + e_- \rightleftharpoons H + \gamma$)

From equation (10) and expressing μ_p and μ_e as functions of n_p and n_e like we did to express equation (11), the number density of hydrigen is :

$$n_H = \left(\frac{g_H}{g_p g_e}\right) n_p n_e \left(\frac{m_e T}{2\pi}\right)^{-\frac{3}{2}} e^{\frac{B}{T}} \tag{19}$$

We notice that the number density of baryon is such as $n_b = n_p + n_H$. By definition, the fractional ionization of the electron is $\alpha_e = \frac{n_e}{n_b} = \frac{n_p}{n_b}$. The resolution of equation (8) for photons gives $n_\gamma = \frac{2\zeta(3)}{H^2}T^3$. We can estimate n_b because of the relation $\frac{n_b}{n_\gamma} = \eta(\Omega_b h^2)$ where $\eta = 2.739 \cdot 10^{-8}$ (*Spergel et al, 2007*) and $\Omega_b h^2 = 0.0222$ (*Molaro, 2007*). Finally, considering $H^2 \equiv \pi^2 \left(T^3\right)^{\frac{3}{2}}$ as in (*Padmanabhan*, 1993), equation (19) becomes :

$$\frac{1-\alpha_e}{\alpha_e^2} = \frac{4\sqrt{2}}{\sqrt{\pi}} \cdot \zeta(3)\eta\Omega_b h^2 \left(\frac{T}{m_e}\right)^{\frac{3}{2}} e^{\frac{B}{T}} = 2.33 \cdot 10^{-9} \left(\frac{T}{m_e}\right)^{\frac{3}{2}} e^{\frac{B}{T}} \tag{20}$$

The fractional ionization stages the percentage of free electrons in the plasma. For some percentage, we estimated the corresponding time and temperature (*cf* Fig 6). We observe a threshold value, for $T \sim 0.32$ eV, for which most of the electrons seem to be binded with protons to from hydrogen atoms.

α_e	Time	Temperature	Energy
10%	250 000 years	4137 K	0.356 eV
5%	270 000 years	3983 K	0.343 eV
2.5%	290 000 years	3843 K	0.331 eV
1%	320 000 years	3673 K	0.317 eV
0.5%	340 000 years	3555 K	0.306 eV

FIG. 6 – First atoms of hydrogen

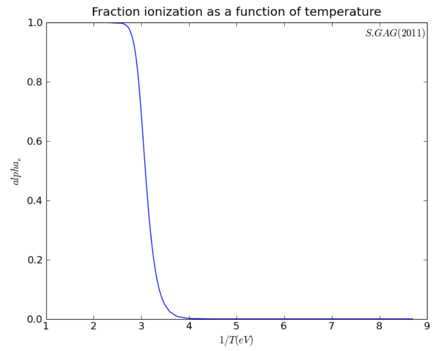

FIG. 7 – Evolution of fraction ionization with temperature.

The temperature for which most of the atoms of hydrogen were formed tend to be indicated as being the recombination temperature (*cf Appendix B - A brief history of the universe*).

The deuterium bottleneck.

At the begining of the Big Bang Nucleosynthesis, the high temperature and the high density of photons induced photons to be extremely energetic. That being, no proton can associate with a neutron long enough to form deuterium. Indeed, at that temperature, the binding energy of deuterium is lower that the average kinetic energy of a photon. The density of photons being important, so are their average kinetic energy, any deuterium formed is almost immediately broken by a photon. This situation is known as the *deuterium bottleneck*. The interest on the production of deuterium comes from the fact that this isotope of hydrogen is a required step to form ^{4}Helium. As temperature decreases with time, the average kinetic energy per photon reaches the binding energy of deuterium. The temperature decreases, so does the density of photons and therefore the probability of a photon to interact with deuterium. At that point, deuterium cannot be destroyed by a photon because they are not energetic enough to break the proton/neutron association. There is a short window during which the temperature is low enough to allow the deuterium, and then the ^{4}Helium, to form and high enough so that nuclear reaction keep occuring. Once this window has passed, nuclear reactions stop and the elemental abundances are fixed. They will only change because of some other processes involving nuclear reaction occuring in the rest of the universe evolution.

Photons decoupling : the last scattering surface.

Even though radiation and matter maintained a good thermal contact bacause of the intercation between photons and electrons through Thomson scattering, things changed when electrons decoupled. The density of free electrons became too low to maintain photons is thermal equilibrium and soon, the free path of photons became larger than the Hubble distance. The interaction rate of photons is given by : $\Gamma_\gamma = n_e\sigma_T$ where $\sigma_T = 6.65 \cdot 10^{-25}\, cm^2 = 1542.3\, GeV^{-2}$. Simplifying equation (10) for the electron case, we have : $n_e(T) = 2\left(\frac{m_e T}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m_e}{T}}$. The numerical solving of $\Gamma_\gamma = H(T)$ gives us the moment when photons decoupled, also called the last scattering surface.

Other atoms.

We apply the same methods to evaluate the abundances of light elements within a range of mass from the tritium up to the lithium. The predictions of abundances have all been verified except for the the lithium. It is refered as the lithium problem. Please refer to (*Cyburt, 2008*) for more details on that question.

2. Dark Matter abundance

2.1. The Boltzmann equation

All chemical elements that we are now aware of result, directly or not, from the production of light species through nuclear reactions at the recombination[9]. The Boltzmann equation relates the rate of change in the abundance of a given species to the difference between the rates for producing and eliminating the species involved in the reaction. It is writen as :

$$\underbrace{\underbrace{\underbrace{\frac{dn}{dt} = \langle\sigma v\rangle(-n^2}_{\text{(*) first regime at equilibrium}} + n_{eq}^2)}_{\text{(**) second regime at equilibrium}} - 3Hn}_{\text{(***) out of equilibrium}} \tag{21}$$

From the equation above, we can make the following comments :

- $\frac{dn}{dt} \equiv$ the evolution through time of the number density $[m^{-3} \cdot s^{-1}]$
- $\langle\sigma v\rangle =$ cross section $\equiv$ the likelihood of intercation $[10^{-4}m^3 \cdot s^{-1}]$[10]
- $-\langle\sigma v\rangle n^2 \equiv$ the number of particle annihilated per unit of time per unit of volume $[10^{-4}m^{-3} \cdot s^{-1}]$
- $\Gamma = \langle\sigma v\rangle n \equiv$ the interaction rate per particle for the considered interaction. The units of Γ are the inverse of time. $[10^{-4}s^{-1}]$
- $\langle\sigma v\rangle n_{eq}^2 \equiv$ the number of particle produced per unit of time per unit of volume [11] $[10^{-4}m^{-3} \cdot s^{-1}]$
- $-3Hn \equiv$ can be seen as the diluted number density, where H is the Hubble constant $[km \cdot s^{-1} \cdot Mpc^{-1}]$

- When solving (*) and (**), one can see that the dominant term of the solutions are, respectively, $n \sim T^{-\frac{3}{2}}$ and $n \sim e^{-\frac{m}{T}}$. As for (***), it requires a numerical solving.

We can notice that it is because of (***) that, today, we can observe non negligible abundance of species. Indeed, if the evolution of the number density had only been the two regimes at equilibrium, then the exponential factor, also called the Boltzmann factor, would have turned the number density of the considered species near zero.

For a given species, when stepping from the phase (**) to the phase (***), the universe leaves an epoch where $\Gamma \geq H$, meaning that there is more interacting than expanding, hence the species is coupled, to enter an epoch where $\Gamma \leq H$, meaning that there is more expanding than interacting. Since the interactions stop, there is no more production of the species considered, the abundance *freeze out* and the species decouples. The temperature for wich the freeze out occurs is given by $\Gamma = H$. We have seen an illustration earlier with the Fig (5).

2.2. Number density at equilibrium

The Special Theory of Relativity tells us that there is an equivalence between mass and energy. Therefore, when applied to the radiation dominated era, we can consider that particles are continuously undergoing reactions in which they annihilate and produce each other (photons $\leftrightarrow$ particles/antiparticles). If so, one can say that the matter and the radiation are in thermal equilibrium because they can freely convert back and forth.

We are defining new quantites. First, the density number per unit of entropy : $Y = \frac{n}{T^3} \simeq \frac{n}{s}$. Then, $x = \frac{m}{T}$.

At equilibrium for a non relativistic fermion or boson ($x \gg 3$) :

[9] Appendix A gives a brief view of the different epochs in the history of the universe.
[10] σ is in $cm^2 \equiv 10^{-4}m^2$.
[11] The index $_{eq}$ means that the quantity is considered for thermal equilibrium.

$$Y_{eq} = \frac{45}{2\pi^4}\left(\frac{\pi}{8}\right)^{\frac{1}{2}} \frac{g}{g_* s} x^{\frac{3}{2}} e^{-x} = 0.145 \cdot \frac{g}{g_* s} x^{\frac{3}{2}} e^{-x} \quad (22)$$

From (*Kolb & Turner*, 1990), we know that for a relativistic boson ($x \ll 3$) :

$$Y_{eq} = \frac{45}{2\pi^4}\zeta(3)\frac{g}{g_* s} = 0.278 \cdot \frac{g}{g_* s} \quad (23)$$

For a relativistic fermion ($x \ll 3$) :

$$Y_{eq} = \frac{45}{2\pi^4}\zeta(3)\frac{\frac{3}{4}g}{g_* s} = 0.209 \cdot \frac{g}{g_* s} \quad (24)$$

Where $g_* s = \sum_{i=bosons} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8}\sum_{i=fermions} g_i \left(\frac{T_i}{T}\right)^3$ is the effective degree of freedom of all particles (including the relativistic ones).

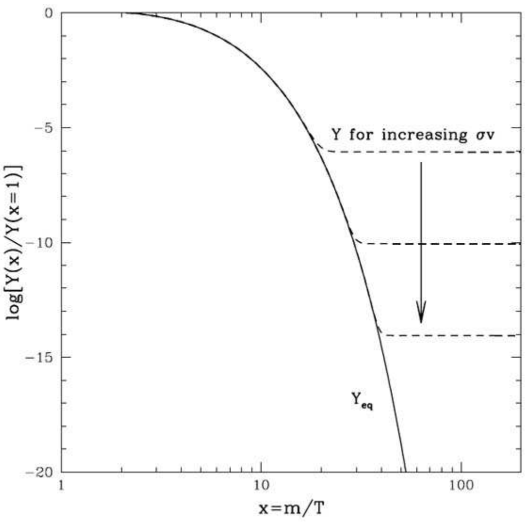

FIG. 8 – Freeze out. *The continuous line is the abundance at equilibrium. The dashed lines are today abundances.*

Let's consider the $(\chi)+(\chi) \rightleftharpoons (SM)+(SM)$ reaction : the conversion of standard model particles into dark matter particles. Because of $\frac{dx}{dt} = -m\frac{\dot{T}}{T} = -xH$, the equation (21) becomes :

$$\frac{dY}{dx} = \frac{\lambda}{x^2}\left(Y_{SM}^2 - Y^2\right) \quad (25)$$

Where $\lambda = \frac{m^3\langle\sigma v\rangle}{H}$.

When solving numerically this equation, we find that Y departs from Y_{eq} around $x \simeq 10$. By integration of equation (25), we have :

$$\int_{Y_f}^{Y_\infty} \frac{dY}{Y^2} \simeq -\int_{x_f}^{\infty} \frac{\lambda dx}{x^2} \quad (26)$$

$$\Rightarrow \ Y_\infty \equiv \frac{x_f}{\lambda} \ \text{ because in radiation dominated era } Y_\infty \ll Y_f \quad (27)$$

Then, a good approximation for relic abundance is : $Y_\infty \simeq \frac{10}{\lambda}$.

The smaller the value of λ, the sooner happens the freeze out. Indeed :

$$\lambda \searrow \equiv \begin{cases} m \searrow \ \text{light particles decouples sooner than heavy particles} \\ H \nearrow \ \text{the sooner particles decouple, the bigger is the value of the Hubble parameter} \\ \langle\sigma v\rangle \searrow \ \text{for fixed conditions, light particles are less likely to interact than heavy particles} \end{cases} \quad (28)$$

We can link that to the observed density, $\Omega_\chi = \frac{\rho_\chi}{\rho_c}$, using :

$$Y_\chi = \frac{n_\chi}{T^3} = \frac{\#_\chi \rho_\chi}{m_\chi T^3} = \frac{10}{\lambda}\frac{\rho_c}{\rho_c} = Y_\infty \quad (29)$$

Therefore :

$$\Omega_\chi = \frac{10 T^3 H}{\#_\chi m_\chi^2 \langle\sigma v\rangle} \quad (30)$$

3. Dark Matter detection

3.1. Possible detection methods

There is two type of detection methods : the direct one and the indirect one. The first method consists in detectors looking for direct interaction with dark matter particles. The second method consists on detecting the products or secondary products of a $(\chi) + (\chi) \rightleftharpoons (SM) + (SM)$ reaction. Appendix C presents with more detail some of the experiments of direct and indirect detection. In this part, we are going to take an interest in the reaction that has electrons and positrons as a secondary product, cf Fig (9).

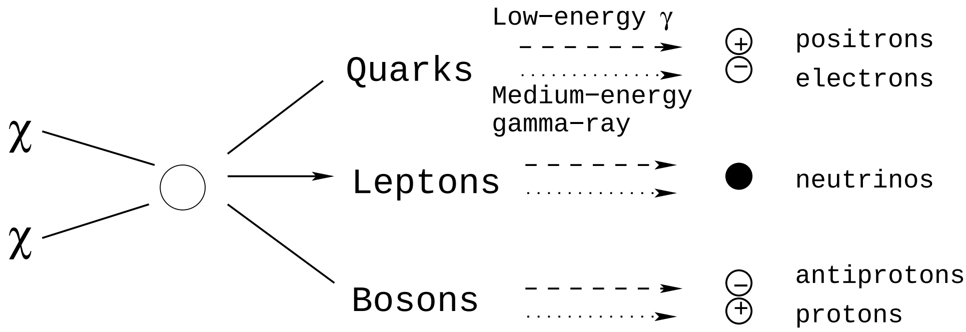

FIG. 9 – Indirect detection.

We are assuming that most of the dark matter is located near the galactic center. This is one of the places where the search of signature of dark matter annihilation has been focused. Indeed, because of the rotation motion at the galactic center, positrons[12] resulting from a dark matter anihilation reaction are accelerated and emit light by synchrotron radiation. In the next part, we are going to link the flux of synchrotron light from the galactic center to the mass of dark matter.

3.2. Synchrotron radiation

We are looking for a way to express the energy loss rate as a function of energy when considering phenomena such as synchrotron radiation[13]. To do so, we must introduce some definitions first.

The flux is the amount of energy passing through an area dS in a time dt :

$$dE = \phi dSdt \tag{31}$$

The brightness is the energy crossing dS, lying within a solid angle $d\Omega$ around the normal $\vec{n}$ to the area in a frequency range $d\nu$ by unit time :

$$dE = I_\nu dSdTd\Omega d\nu \tag{32}$$

The effective area, obtained from dS which has a generic orientation $\vec{n}$ is just $\cos\theta dS$, so the flux per given frequency can be expressed as :

$$\phi_\nu = \int I_\nu \cos\theta dS \tag{33}$$

We define the spontaneous emission coefficient j which is the energy emitted per unit time per unit solid angle, per unit volume :

$$dE = jdVd\Omega dt = \underbrace{j_\nu dVd\Omega dtd\nu}_{\text{monochromatic emission}} \tag{34}$$

We note that :

$$I_\nu = \int dsj_\nu \tag{35}$$

[12]Let us recall that electrons are not charged under SU(3).
[13]Please refer to (*Jackson, 1999*) and (*Rybicki & Lightman, 1980*) for complete notes on synchrotron radiation.

where ds is the line of sight element. For an isotropic emitter, or for a random distribution of emitters, the equation (34) becomes by integration :

$$j_\nu = \frac{1}{4\pi} P_\nu \tag{36}$$

P_ν is the radiated power per unit volume, per unit frequency. So, if n_e is the number density of electrons :

$$P_\nu = \frac{n_e}{\nu} P(E) \tag{37}$$

We now need to compute the radiated power $P(E)$. To do so, we use the fact that a synchrotron emission is the relativistic limit of the cyclotron radiation. The Larmor formula gives the total power radiated by a nonrelativistic point charge as it accelerates :

$$P_{Larmor} = \frac{e^2}{6\pi\epsilon_0 c^3} \mid \vec{a} \mid^2 \tag{38}$$

The expression of motion allows us to express the Larmor formula as a function of magnetic field :

$$m\frac{d\vec{v}}{dt} = e\vec{v} \wedge \vec{B} \ \Rightarrow \ v_{||} = \text{constant};\ \dot{v}_\perp = a_\perp = \frac{ev_\perp}{m} \tag{39}$$

When implementing equation (39) in equation (38) :

$$P_{Larmor} = \frac{e^4 B^2 v_\perp^2}{6\pi\epsilon_0 m^2 c^3} \tag{40}$$

Let's now consider very relativistic positrons. In special relativity, the Lorentz transformation on accelerations gives[14] :

$$P \simeq \frac{e^4 E^2 B^2}{m^4} \tag{41}$$

Now that we have computed the power as a function of the electromagnetic field, we need to apply what was relevant for one charged particule to the flux of particle coming from the galactic center. The motion of positrons coming from the galactic center is given by the diffusion equation :

$$\underbrace{\frac{\partial}{\partial t}\frac{dn_e}{dE_e}}_{\text{spectrum variation in time}} = \underbrace{\vec{\nabla}\cdot\left[\kappa(E_e,\vec{r})\cdot\vec{\nabla}\frac{dn_e}{dE_e}\right]}_{\text{diffusion}} + \underbrace{\frac{\partial}{\partial E_e}\left[b(E_e,\vec{r})\frac{dn_e}{dE_e}\right]}_{\text{energy loss}} + \underbrace{Q(E_e,\vec{r})}_{\text{source}} \tag{42}$$

We are now going to make the following assumptions :

– because of thermal equilibrium, we suppose that creation and loss rate have the same order of magnitude : $\frac{\partial}{\partial t}\frac{dn_e}{dE_e} = 0$
– in the inner part of the galaxy, we neglect the diffusion : $\vec{\nabla}\cdot\left[\kappa(E_e,\vec{r})\cdot\vec{\nabla}\frac{dn_e}{dE_e}\right] = 0$

Hence we are left with :

$$Q(E_e,\vec{r}) + \frac{\partial}{\partial E_e}\left[b(E_e,\vec{r})\frac{dn_e}{dE_e}\right] = 0 \tag{43}$$

From equation (43), the spectrum is thus given by :

$$\frac{dn_e}{dE_e} = \frac{\tau}{E_e}\int_{E_e}^{m_\chi c^2} dE'_e, Q(E'_e,\vec{r}) \text{ where the time scale is : } \tau = \frac{E_e}{b(E_e,\vec{r})} \tag{44}$$

Plus, the source term is, by construction :

$$Q = \langle\sigma v\rangle \frac{dN_e}{dE_e}\left[\frac{\rho_\chi}{m_\chi}\right]^2 \tag{45}$$

Let us now compute the energy loss rate. Near the galactic center, we neglect the Compton radiation and the Bremsstrahlung radiation and we keep the following terms : $b(E_e,\vec{r}) = P_{synchr} + P_{ICS}$. Where P_{synchr} is the Synchrotron radiation power, and P_{ICS} is the Inverse Compton scattering power. Those terms can be expressed as :

$$P_{synchr} \simeq E^2 B^2 \tag{46}$$

[14]We are taking back the notations $c = \hbar = 1$.

$$P_{ICS} \simeq E^2 U_{rad}(\vec{r})(\propto \sigma_T) \tag{47}$$

$U_{rad}(\vec{r})$ has the same dimension that an energy density.

Therefore the energy loss rate is :

$$b(E_e, \vec{r}) = P_{synchr}\left(1 + \frac{U_{rad}}{B^2}\right) \tag{48}$$

Now that we have everything, let's compute the flux. The synchrotron radiation energy density per unit time and unit frequency is :

$$L_\nu(\vec{r}) = \int P_\nu(E)\frac{dn}{dE}(E, \vec{r})dE \tag{49}$$

Hence, the energy flux by unit frequency is :

$$J_\nu = \frac{1}{4\pi}\int_{\text{line of sight}} d\vec{l} L_\nu(\vec{l}) \tag{50}$$

$$= \int dE d\vec{l} \frac{\langle\sigma v\rangle}{8\pi}\left(\frac{\rho_\chi}{m_\chi}\right)^2 \left(\frac{e^4 E^2 B^2}{m_e^4}\right)^{-1} \frac{1}{b(E_e, \vec{r})\frac{e^3 B}{m_e}\phi\left(\frac{\nu}{nu_c}\right)} \tag{51}$$

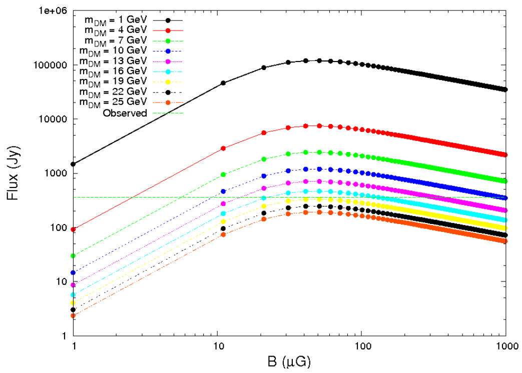

FIG. 10 – Energy flux as a function of magnetic field for different light dark matter masses.

As we can see on Fig (10), when plotting the flux as a function of the magnetic field[a], one can observe the followong behaviour :

$$J_\nu \equiv \frac{B}{B^2 + \#} \tag{52}$$

The flux reaches a maximum around $B \simeq 30\,\mu G$. Indeed, for lower magnetic field positrons emit a weak radiation. For magnetic field stronger than $30\,\mu G$, positrons radiate more than before but there is an energy loss because of different physical processes (ICS...).

It is also interesting to notice that $\frac{dn}{dE} \sim \frac{1}{m_\chi^2}$. Hence, the heavier m_χ, the smaller $\frac{dn}{dE}$, the smaller is J_ν.

[a]Following the Boehm model of light dark matter, *cf* (*Boehm, 2003*), (*Boehm, 2007*) and (*Boehm, 2010*).

† Thank you to Bryan Zaldivar Montero for Fig (10).

When taking a look at the equation (51), we notice that we can now constrain the mass of a particle of dark matter : the flux is an observable, the magnetic field is modelised quite accuratly near the galactic center and an estimation of Ω_χ has been made through different observational cosmology experiments.

Conclusion

After familiarising ourselves with the problem of abundances in the early universe and how we can derive today's abundances thanks to the Boltzmann equation, we have seen how the synchrotron radiation can be used as a method to constrain the mass of dark matter particles. A step further is to overlap the results of different dark matter detection experiments to constrain more efficiently m_χ. That has been done with the LEP and Tevatron analysis in (*Mambrini & Zaldivar, 2011*). Overlapping other analysis from different dark matter search experiments and the results of the upcoming months of Higgs search at LHC are crucial in the "Beyond the Standard Model" field.

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I would like to thank Bryan, Julio, Antoine and Maxime, whom I shared an office with, for the nice work environment they provided.

Finally, I thank Henk Hilhorst for his allowing me to do an internship in his laboratory.

[15]Weekly review of relevant article in theoretical particle physics and cosmology, most often dark matter related.

[16]Séminaire INformel des JEunes.

APPENDIX A - INTRODUCTION

◇ **The Cosmic Microwave Background.**

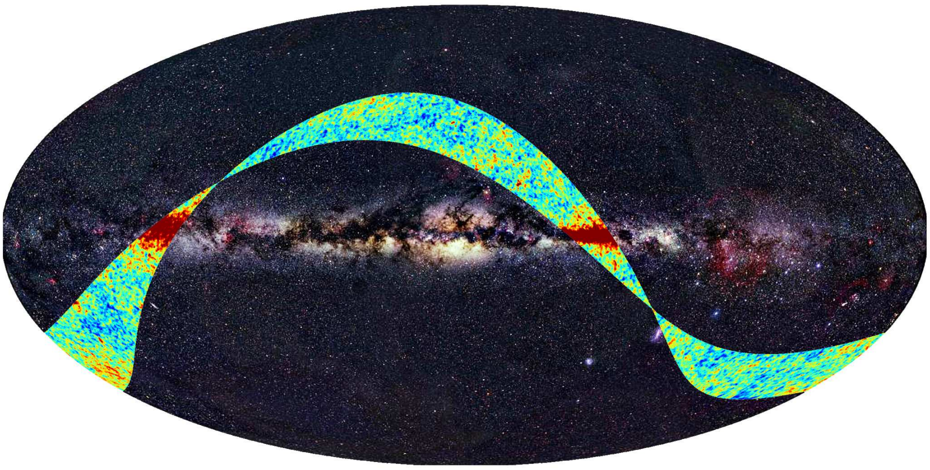

FIG. 11 – First map of a strip of the sky by PLANCK satellite after 2 weeks of continuous sky survey.

◇ **The Bullet Cluster.**

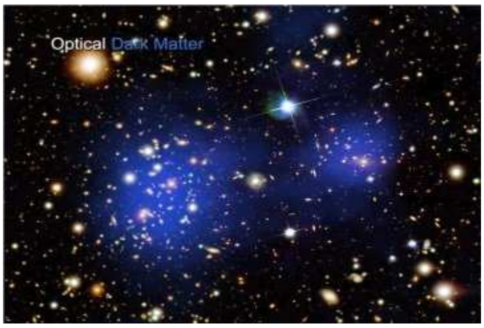

FIG. 12 – Dark Matter position in the Bullet cluster.

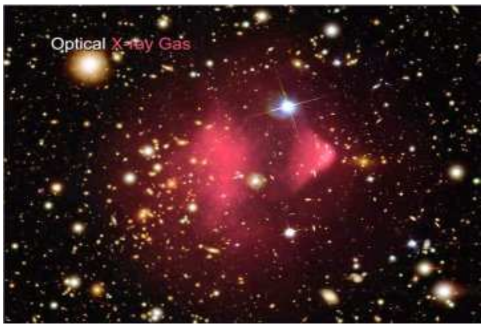

FIG. 13 – Gas position in the Bullet cluster.

◇ **History of the universe.**

Epoch/Event	Time	Temperature	Energy	Redshift
Hot Big Bang	-	-	-	-
Inflation	-	-	-	-
Reheating	-	$T = 0$ K *end of reheating*	-	$z \simeq 10^{28}$ $z^{\text{reheating ends}}$
-Radiation domination era-	-	-	-	
Baryongenesis-Leptogenesis	10^{-4} s	$2 \cdot 10^{12}$	200 MeV	$z = 10^{11}$
Primordial Nucleosynthesis	100 s	10^9 K	0.1 MeV	$z = 10^9$
-Matter domination era-	100 000 years	9000 K	0.81 eV	$z = 3400$
Recombination	380 000 years	3000 K	0.3 eV	$z = 1100$
Reionization	$3.2 \cdot 10^8$ years	120 K	0.01 eV	$z = 20$
First galaxies formation	$1.3 \cdot 10^9$ years	60 K	$5 \cdot 10^{-3}$ eV	$z = 10$
-Dark Energy domination era-	$9.4 \cdot 10^9$ years	4.8 K	$4 \cdot 10^{-4}$ eV	$z = 0.75$
Today	$15 \cdot 10^9$ years	2.7 K	$2.35 \cdot 10^{-4}$ eV	$z = 0$

FIG. 14 – Brief history of the universe

◇ **Decoupling : sum up.**

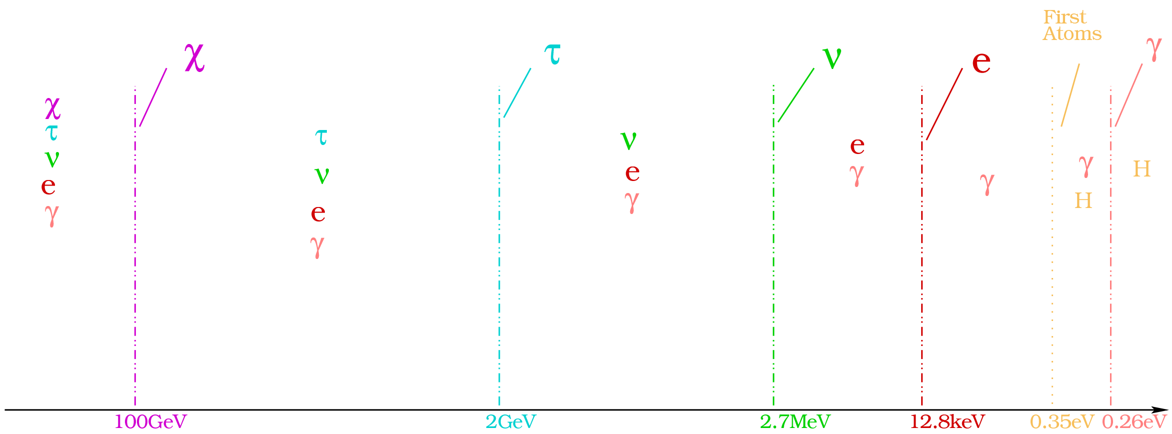

FIG. 15 – Timeline for decoupling.

◇ **Usefull data for light element abundances calculation.**

	Binding energy/nucleon	Total binding energy	Mass
Electron	-	-	0.511 keV
Proton	-	-	938.3 MeV
Neutron	-	-	939.6 MeV
Hydrogen	-	-	938.3 MeV
Deuterium	1.112 MeV	2.224 MeV	1876.1 MeV
Tritium	2.827 MeV	8.482 MeV	2809.4 MeV
Helium3	2.573 MeV	7.718 MeV	2809.4 MeV
Helium4	7.074 MeV	28.3 MeV	3728.4 MeV
Lithium6	5.332 MeV	32.0 MeV	5603.1 MeV
Lithium7	5.606 MeV	37.3 MeV	5603.1 MeV

FIG. 16 – Particles-Species related data

◇ **Evolution of degree of liberty with temperature.**

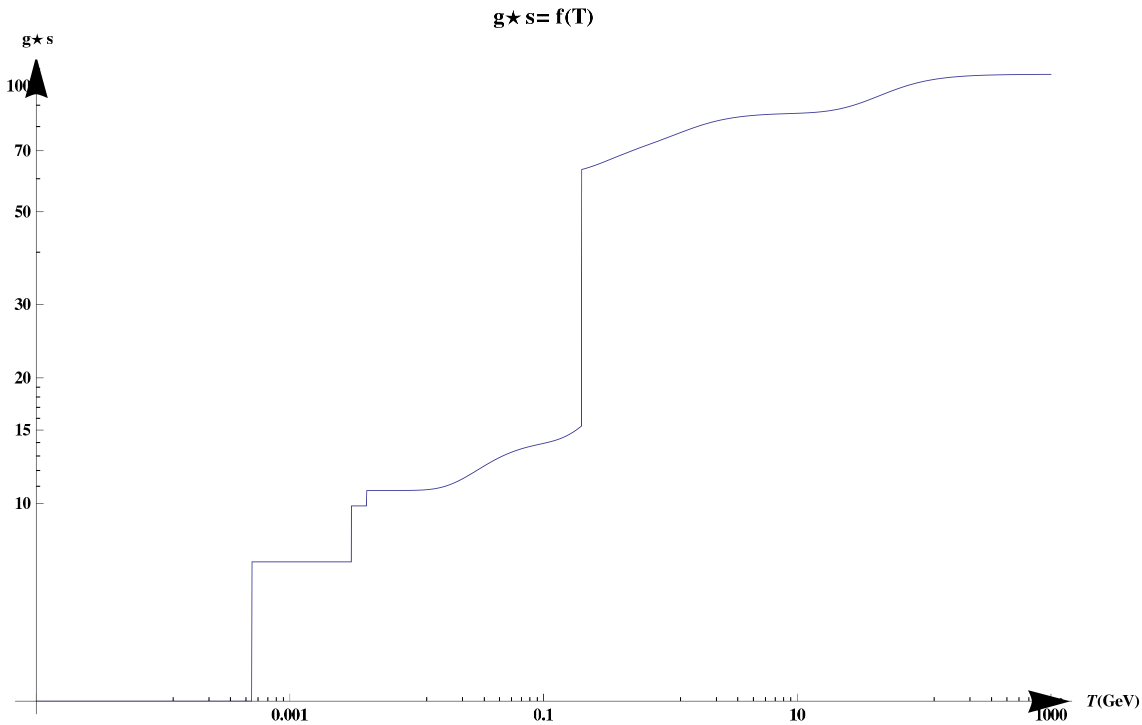

FIG. 17 – g_* as a function of temperature.

† I would like to thank Maxime Follin for his help regarding the part dealing with the degree of liberty and his allowing me to use his mathematica code.

APPENDIX C - Dark Matter detection

◇ **Direct Detection.**

XENON : " XENON100 is a new dark matter search experiment, aiming to increase the fiducial liquid xenon target mass to 100 kg with a 100 times reduction in background rate, compared to the XENON10 experiment which used liquid xenon as a sensitive detector medium to search for WIMPs (Weakly Interacting Massive Particles)."
▹ http ://xenon.astro.columbia.edu/index.html

CoGeNT : "The CoGeNT Dark Matter Experiment is a direct search for signals from interactions of dark matter particles in a low-background germanium detector located at Soudan Underground Laboratory in Soudan, MN."
▹ http ://cogent.pnnl.gov/

Other direct search experiments : COUPP, CDMS, DAMA, EDELWEISS, CRESST, EURECA, ZEPLIN, DEAP, ArDM, WARP, LUX, SIMPLE, PICASSO, DMTPC, DRIFT, KIMS...

◇ **Indirect Detection.**

PAMELA : "Among the most plausible candidates for dark matter are weakly interacting massive particles (WIMP), of which the supersimmetric neutralino is a favourite candidate from the point of view of particle physics.Neutralino are Majorana fermions and will annihilate with each other in the halo, resulting in the symmetric production of particles and antiparticles, the latter providing an observable signature. The Pamela Instrument is installed on the up-ward side of the Resurs-DK1 satellite and has been launched the 15th of June 2006 from the launch site of Bajkonour in Kazakhstan by a rocket Soyuz. With Pamela we look for annihilations that produce antiprotons and positrons."
▹ http ://pamela.roma2.infn.it/index.php

Other indirect search experiments : WMAP,FERMI/GLAST, ATIC, VERITAS, HESS, IceCube, AMS...

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